Nautical Science

Poor

By CHARLES LANE POOR

THE SOLAR SYSTEM

A Study of Recent Observations

NAUTICAL SCIENCE IN ITS RELATION TO PRACTICAL NAVIGATION

Together with a Study of the Tides and Tidal Currents

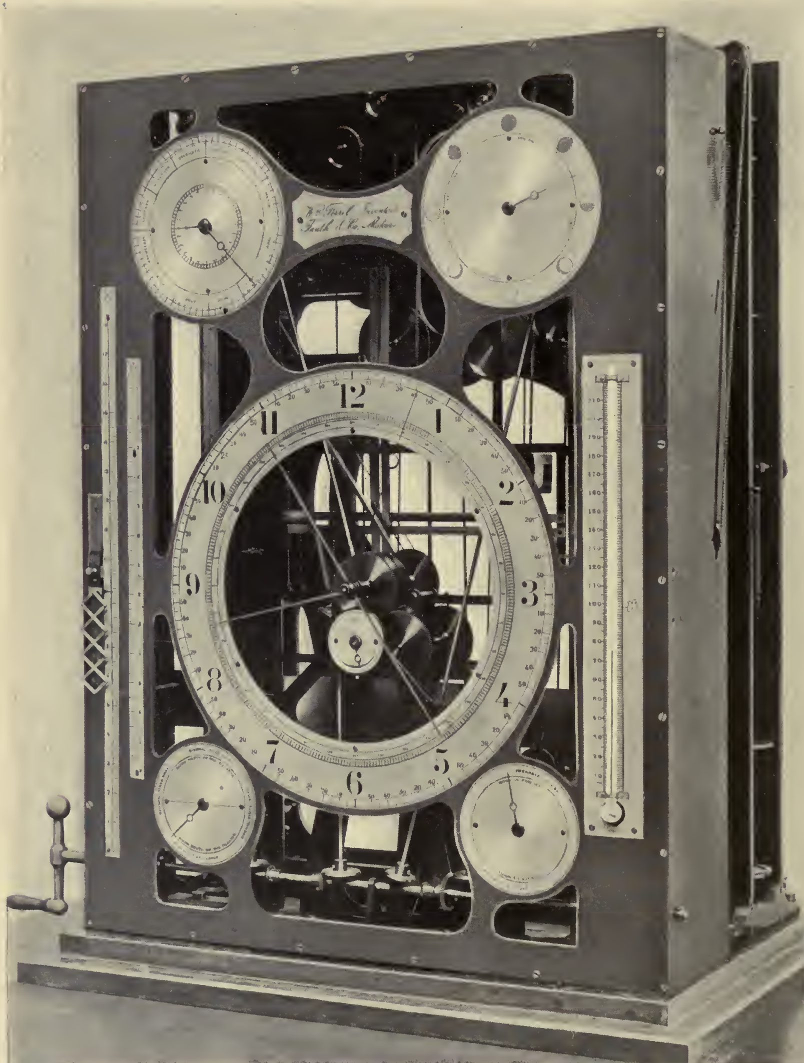

PLATE X

The Tide-predicting Machine of the Coast and Geodetic Survey. (*Frontispiece*)

NAUTICAL SCIENCE

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TOGETHER WITH A STUDY OF THE
TIDES AND TIDAL CURRENTS

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Professor of Astronomy in Columbia University, Author of "The Solar System," etc.

ILLUSTRATED

G. P. PUTNAM'S SONS
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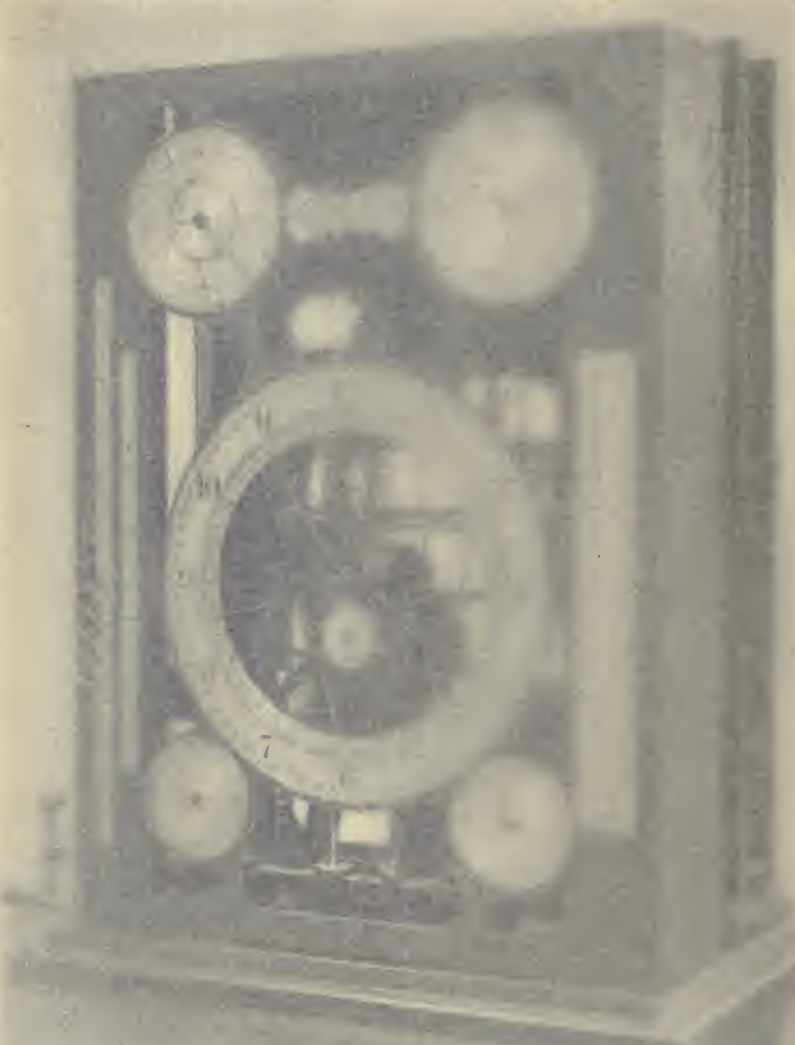

PLATE X

[illegible] of the Caius and Gonville Library. [Frontispiece

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PREFACE

THIS work is intended for the general reader as well as for the practical navigator. It is an attempt to explain in non-technical language and without the use of complicated mathematical formulas the fundamental facts and principles that form the basis of all navigational methods. Navigation is founded upon Astronomy, but it is not essential for the navigator, in order to find his way from port to port, to know the methods by which the size, the shape, and the motions of the earth are determined, and to be familiar with the physical characteristics of the sun and stars. A knowledge of these astronomical facts is, however, necessary to those who wish to be thoroughly posted in the science of navigation. Every navigator knows how to use the data found in the *Nautical Almanac*, but few have the slightest idea how these data are collected and made available for the immediate needs of the practical sailor.

The Sumner method of finding one's position at sea is fundamental and, when combined with modern methods of reduction, is most simple and readily applied to all navigational problems.

The time-honoured noon sight for latitude and the morning or afternoon sight for longitude are but special cases of this most powerful method. The theory of the Sumner lines of position is so easy to understand, and at the same time it is so widely applicable, that it is made the basis upon which the whole theory of practical navigation depends. At the end of each chapter is to be found a sort of appendix containing notes, formulas, and practical examples. This portion of the book forms a condensed treatise on modern methods of navigation.

A considerable portion of the book is devoted to an explanation of the tides and tidal currents; their peculiarities and their causes. That the tides are caused by the attraction of the sun and moon is well known, but just how this attraction causes the radically different tides in different portions of the earth is not fully realised. Until very recently the ablest investigators considered the tides as world phenomena, the tides in each bay and ocean as a part of one great tidal wave which sweeps around the entire world. To-day the able researches of Dr. Harris have shown us that the tides and tidal currents are essentially local phenomena, that the tides of each ocean basin are practically independent of those of the rest of the world.

Without the hearty co-operation and assistance of the Coast and Geodetic Survey this portion of the book could never have been put into

its present shape. Mr. Otto H. Tittmann, the Superintendent, kindly placed the facilities of the Survey at the author's disposal and furnished maps, charts, and data, which have been freely used for illustration. Dr. Rollin A. Harris, whose tidal theories the author has attempted to explain in non-technical language, kindly read and revised the manuscript. To him are due many valuable suggestions as to text and to illustrations.

The thanks of the author are also due to Professor Geo. E. Hale, Director of the Carnegie Solar Observatory, to Professor Edwin B. Frost, Director of the Yerkes Observatory, to Mr. John Bishop Putnam, and to others, for kindness in furnishing original photographs and drawings used to illustrate the text; to Dr. M. F. Weinrich, who read and corrected the proof sheets and compiled the index.

C. L. P.

October, 1909.

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Nautical Science

CHAPTER I

THE EARTH AS AN ASTRONOMICAL BODY

TWO thousand years ago were the words written, "If at sea we sail towards mountains or other high objects, we see these objects rise from the sea, where they have been concealed hitherto by the curvature of the surface of the water." Other and even better proofs of the spherical shape of the earth were given by the astronomers of old. They knew that the earth and moon are opaque bodies, that the moon shines by the reflected light of the sun, and that a lunar eclipse is caused by the moon passing into the shadow cast by the earth. Now they noticed that the outline of the earth's shadow, thus cast upon the moon, was always a smooth curve, the arc of a circle; and they knew that a sphere is the only body that always casts a circular shadow. A flat disc, a hemisphere, or an

irregularly shaped body may, when placed in some special position, cast a circular shadow: but let another side of such a body be turned toward the light and the shadow will show straight lines and sharp, irregular edges.

Thus from prehistoric times it was well known that the earth is a globe. The popular idea that Columbus discovered this fact, and that it was this discovery which led to his voyage to America, is utterly wrong. At the time of Columbus it was only the unlearned and the intentionally ignorant who did not know that the earth is spherical. In fact the earth was measured some seventeen centuries before Columbus was born. Eratosthenes, about the year 200 B.C., measured the circumference of the earth and found it to be about 250,000 stadia, or some 30,000 English miles. This is some twenty per cent. too great, for according to the latest measures, the equatorial circumference of the earth is slightly less than 25,000 miles.

In making such a determination of the earth's size, it is only necessary actually to measure a small portion of the circumference. One degree is one three hundred and sixtieth part of a circumference, and if the distance in feet and miles between two points just one degree apart be found, then will the entire circumference of the earth be just three hundred and sixty times the distance between these two points. It is not essential that the two points be just one degree apart;

they may be at any convenient angular distance, the greater the better. Thus the actual problem of measuring the earth consists of two distinct operations; namely:

1. The measurement of the angular distance in degrees, minutes, and seconds between two points on the earth's surface: this angular distance is usually the difference in latitude between the two chosen places.

2. The measurement of the actual distance along the surface of the earth in miles, feet, and inches between the same two places.

There is no great difficulty in the first of these operations. Astronomical instruments and methods are now so accurate that the latitude of a place can readily be determined to within one tenth of a second of arc, or to within $\frac{1}{12,960,000}$ part of the circumference. To obtain this degree of accuracy, carefully built and mounted instruments are necessary, but the astronomical methods by which the latitude is determined do not differ essentially from those used at sea. And these methods are fully outlined and explained in Chapter VIII.

Far different, however, is the second part of the problem, the accurate measurement of the distance between the two stations selected as the ends of the arc. From the time of Eratosthenes this portion of the problem has presented great difficulties. The surface of the earth is rough and is traversed by valleys, by hills, and by rivers.

Over such an uneven surface the determination, in feet and inches, of the distance apart of two marks, separated by even a few miles, becomes extremely laborious and consumes much time and patience; the actual physical measurement of great distances becomes impossible. Such distances are now determined by means of an elaborate survey and triangulation.

The two points, whose distance apart is required, are connected by a series of triangles, the corners of which are marked by elaborate posts or signals. At each station, thus marked, the angles of the various triangles meeting at that point are carefully measured. Thus are found the three angles of each and every triangle of the series. Now, if the angles of a triangle and one side are known, the lengths of the other two sides can be found by simple calculations. Hence if but one side of the first triangle be measured, the lengths of the other two sides can be found from this and the measured angles. One of these two sides is also a side of the second triangle, and from this known length and the angles the lengths of the other sides can readily be found. Similarly for the third triangle, one of its sides is also a side of the second, and is thus known from calculation. From the measured length of one side of the first triangle are thus calculated the lengths of the sides of all the triangles and finally the required distance between the two places. The side which is actually

measured is usually comparatively short and is the so-called "base-line."

For measuring the base-line metal rods or bars of known lengths are used. These are placed end to end and the length of the base-line measured off just exactly as one would measure the length of a room with two yardsticks, always picking up the back one and laying it down in front of and just touching the forward end of the other. Great care must be exercised in having the rods exactly level and pointing in a straight line. For this reason the path along which the measurement is made is carefully prepared and levelled beforehand. The difficulty of the operation is considerably increased by the fact that the rods change their lengths with the varying temperatures to which they are of necessity exposed, when taken into the open fields. Many devices have been used to overcome this temperature difficulty, one of the simplest and best being the Woodward "ice-bar apparatus." In this the metal measuring bar is supported in a trough and completely packed in ice and thus maintained at a uniform temperature of 32° F.

Arcs of the meridian have been measured in various parts of the earth, and the results of these various measures do not agree. In the northern parts of Europe a degree of latitude is nearly 69.4 miles long, whilst near the equator one degree is but 68.7 miles. These show that the earth is not an exact sphere, that it approaches

more nearly to the figure of an ellipsoid. That is, the equator and all the parallels of latitude are almost exact circles, while the meridians, which run from pole to pole, are of a distinct elliptic form. Again the equatorial diameter, as measured from the Atlantic to the Pacific, is some twenty-seven miles longer than the diameter along the axis from pole to pole. This flattening at the poles and bulging out at the equator was probably caused many millions of years ago when the earth was a hot, plastic mass. In fact it can be shown mathematically that a rotating fluid or viscous mass will always become ellipsoidal in shape. This can also be shown experimentally in a number of ways, the simplest of which, perhaps, is by means of a light metal ring, so mounted that it can be rotated with great rapidity about a vertical diameter. When the ring is at rest it is circular in shape, but when it is rotated, it becomes flattened along the axis, bulging out at what we may call the equator. The faster the ring is rotated, the greater and greater becomes its departure from circular shape.

The earth, however, is not an exact ellipsoid. The latest measures indicate that the equator is not a perfect circle and that there are many places where local and continental irregularities cause the actual surface to depart greatly from any known geometrical figure. The equatorial diameter which passes through Ceylon and the Galapagos Islands off the coast of Ecuador is

Plate I

Measuring a Base-line with Iced-bar Apparatus

apparently some 3000 feet shorter than the diameter at right angles, which passes through Liberia and the Gilbert Islands.

Notwithstanding these departures, it is usual to consider the earth as a perfect ellipsoid and to regard such irregularities as altitudes or depressions in the surface. On the whole, the earth is a remarkably smooth globe, and its departure from a spherical form very slight. If a true model of the earth two feet in diameter be made out of well seasoned wood or metal, so as to get a very smooth and polished surface, then the differences in length of the polar and equatorial diameters would be about $\frac{1}{12}$ of an inch, and all the variations in height in the United States, all the mountain ranges and valleys, would be represented in a layer of varnish $\frac{1}{100}$ of an inch thick, which protects the surface.

The figure which has been generally adopted as most nearly representing the actual size and shape of the earth is the ellipsoid as determined by Colonel Clarke, of the English Ordnance Survey, in 1878. According to this determination the earth has the following dimensions:

Equatorial radius	3963.296 miles
Polar radius	3949.790 "

And these show that the polar compression, or difference of the two radii, is only 13.506 miles.

Mass and Density of the Earth

Practically nothing was known about the

interior constitution of the earth until comparatively recent times, and even to-day our knowledge on this important subject is far from what it should be. The first practical measurement, which gave an approximate idea of the density of the earth's interior, was made in Scotland by Maskelyne less than a hundred and fifty years ago. Crude and rough as this determination was, yet it clearly showed the interior of the earth to be solid and more dense than the surface rocks and mountains.

The method used by Maskelyne for measuring the average density of the earth is extremely simple and can readily be explained. To-day, however, there are several more accurate methods but they are more complicated and require delicate and intricate apparatus, and in explaining and studying the apparatus one is apt to lose sight of the essential principles involved. The size and shape of the earth are known with a high degree of accuracy and therefore its cubic capacity can readily be computed. If therefore the total number of tons of matter which it contains can be found, it is a simple example in arithmetic to calculate the average density, or the amount of matter contained in a cubic foot, or a cubic inch.

Now the total amount of matter contained in the earth can be found by weighing it against some body with a known and definite mass. When we wish to know how much matter there is in a barrel of sugar, we weigh, or balance, the sugar

against iron weights whose masses are known and stamped on their faces. In the case of the earth, while we cannot put it into the pan of a scale and weigh it as we do the sugar, yet its attraction for all bodies allows us practically to accomplish this seeming impossibility.

Suppose for a moment the earth to be a perfectly smooth globe and a plumb-line to be hung at any point over its surface, then will the bob be attracted by the earth and point directly towards the earth's centre. For the earth attracts all bodies towards its centre and as though all its mass were concentrated at that point. Now further suppose that a large body, a mountain for example, be placed on the smooth surface of the earth and comparatively near the plumb-line. The matter in the mountain will attract the bob and the plumb-line will no longer point directly towards the centre of the earth, nor will it point directly towards the mountain. All this is shown in the accompanying diagram. Under the influence of the earth alone the plumb-line would take the direction of MA′; under the influence of the mountain alone, the bob would swing around until the line took the direction of MC; under the combined attraction of the two, earth and mountain, the line actually takes the direction of MA: very slightly different from MA′, for the attraction of the mountain is small as compared with that of the earth. From the direction in which the plumb-line points the

relative attractions of the earth and the mountain can thus be determined; and these attractions depend upon the amounts of matter in the earth and the mountain and upon their distances from

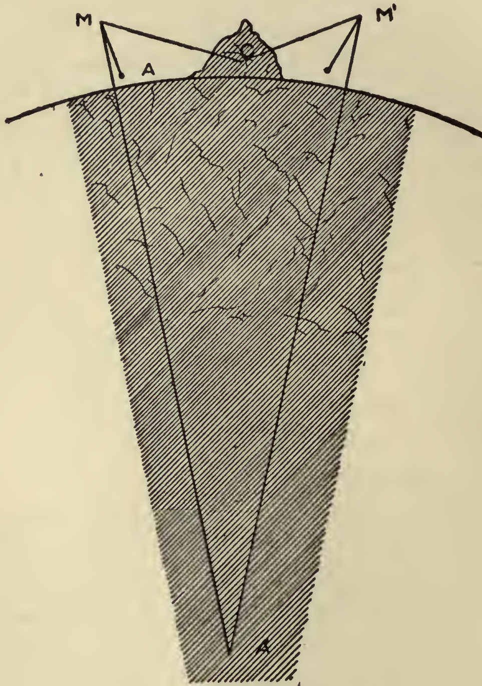

Fig. 1. Weighing the Earth

the bob. These distances can be found, for the radius of the earth is some four thousand miles, and the distance of the bob from the centre of the mountain can be measured. With these distances

known, the relative attractions (found from the direction of the plumb-line) give at once the ratio of the earth's mass to that of the mountain.

In order to find the mass, two elements are essential: a knowledge of the mass of the mountain in tons, and of the deflection of the plumb-line from its normal position. The first of these can be found by means of careful surveys and borings, especially if the mountain be of fairly regular shape. The second can be found by moving the plumb-line across the mountain, from the north to the south side. Here the bob will be deflected towards the north, in a direction opposite to that of the first position. The plumb-lines at M and M′ are thus drawn inward towards the mountain, and the angle between them, which is equal to the difference of latitude of M and M′, will be greater than it would be if there were no mountain there. The difference between these latitudes, as determined from astronomical observations, will be greater than the actual difference as found by measuring the distance between the two points on the surface of the earth. And this difference between the results of observation and measurement is twice the deflection of the plumb-line, caused by the attraction of the mountain.

Maskelyne in 1774 thus found the average density of the earth to be 4½ times that of water. A repetition in 1832 of his measurements at Arthur's Seat, near Edinburgh, gave 5⅓ as a more probable value. Cavendish, by means of delicate

laboratory experiments, found the mean density to be $5\frac{1}{2}$, while very recent experiments by Boys in England and Braun in Bohemia indicate that this figure is a little too low. The results of Boys give a mean density of 5.527.

This high average density gives some indication of the probable condition of the earth's interior. It is well known that the temperature increases toward the centre, the average rise being one degree Fahrenheit for each fifty feet in depth below the surface. If this rate of increase be maintained, then one or two hundred miles below the surface the heat is sufficient to melt the rocks and fuse the metals that form the surface. This in connection with volcanic phenomena caused geologists to consider the interior of the earth as molten and the solid surface crust as comparatively thin. But the melting-point of rock-like substances is raised with pressure; and the pressure in the interior of the earth is enormous. A few yards below the surface this pressure is several times greater than the ordinary atmospheric pressure on the surface; at the depth of a few miles the pressure is measured by tons instead of by pounds. This increase of pressure is so rapid as compared to the increase of temperature, that at no point within the interior is the temperature high enough to melt the substances of which the earth is composed. Physicists and astronomers now believe that the earth is solid throughout, and that it is more rigid than steel. Volcanoes are

mere pockets, mere local phenomena. The tides, the precession of the equinoxes, and the variation of latitude, all take place precisely as though the earth were a solid and extremely rigid body: these phenomena would be utterly changed were the earth a fluid mass, surrounded by a thin solid shell.

Atmosphere

One of the most important parts of the earth, so far as we are concerned at least, is the atmosphere. This, the air, which we breathe and in which we live, is a great envelope of permanent gases and vapours, which surrounds the solid globe. It is just as much a part of the earth, as are the oceans and the mountains; the land, the waters, and the air form one body, which rotates as a whole, and which travels through space as a whole. Ptolemy nearly two thousand years ago failed to grasp this fact, and this failure on his part delayed the progress of astronomy for fifteen centuries. He clearly saw that, if the alternation from day to night is caused by a rotation of the earth, then points on the equator must move with a speed of nearly 1000 miles an hour. If the atmosphere be at rest and the solid earth rotates within it at this terrific speed, then a steady gale, exceeding in severity more than tenfold the strongest hurricane, would always blow from the east: birds in flight and objects thrown into

the air would be left behind and carried with frightful rapidity towards the west, trees would be torn up by the roots, and houses and cities levelled to the dust. As these things do not occur Ptolemy concluded, not that the air is part of the earth and revolves with it, but that the earth must be at rest. He thought of the earth as immovable and at the very centre of the universe.

Just how thick this atmospheric blanket is no one knows. At its outermost limits it is exceedingly thin and rare, but as the surface of the solid earth is approached, the weight of the outer layers compresses it and it becomes denser and denser. So rapidly does the density of the atmosphere decrease as one goes upward from sea-level, that only the extreme lower portions of the air can be actually explored. The highest altitude ever reached by man was attained by James Glaisher and James Coxwell, who made a balloon ascension in 1862. They reached an altitude of over 37,000 feet, or about seven miles. During the last few years a great many small balloons have been sent up, both here and abroad. These contain automatic self-recording instruments, but carry no living thing. Some of these have risen as much as fifteen miles above the surface. The light of meteors and shooting stars is caused by the rapid flight of these bodies through the upper layers of the atmosphere. The friction caused by their striking against the particles of the air fuses and melts them, and renders them visible.

The heights at which they appear can readily be measured, and many have been observed at altitudes much greater than 100 miles. The atmosphere is, therefore, at least 100 miles deep, and it may and probably does extend much farther.

This atmospheric blanket under which we live introduces a complication into all astronomical observations, into every "sight" of the sun, with which a position at sea is determined. Unless a body be directly overhead, it is never seen in its true direction. The rays of light from the sun or from a star travel through the atmosphere in a curved path; they are "refracted" or bent out of their course, and this bending or refraction is one of the troublesome corrections which bother the astronomer as well as the navigator.

Whenever a ray of light passes from a rarer to a denser medium, its direction is suddenly changed. This is very noticeable when light passes from air into water, or from air into glass, and this principle is utilised in the manufacture of prisms and lenses, in the making of the eye-glass which aids the near-sighted, as well as in the construction of the giant telescopes which bring to our knowledge countless millions of stars. Now the atmosphere, from its upper limits downward, is a medium of gradually increasing density, and a ray of light, therefore, coming from a star to our eye is continually passing from a rarer to a denser medium, and it is, therefore, being continuously bent. Such a ray describes a curved path through the atmosphere

and the direction in which we see the star is that from which the light finally reaches our eye.

The astronomical refraction raises all bodies, it makes them appear higher in the heavens than they really are. It decreases their zenith distance without altering their azimuth. The amount of this apparent lifting changes with the distance of the body above the horizon. At the zenith, or directly overhead, the refraction is practically zero, at the horizon it amounts to nearly two thirds of a degree. The apparent diameters of the sun and moon are only about one half a degree; so that, at the horizon, the refraction is more than the diameter of these bodies. When, therefore, the lower edge of the sun appears just touching the horizon, the entire body is in reality below the horizon and entirely out of the direct line of our vision. We actually see the sun around a "corner" of the earth.

If the air were always perfectly still and never changed its density, the refraction would always be the same for the same altitudes. But the air is never still, it is continually disturbed by currents, and by storms: its temperature, as given by a thermometer, its pressure, as measured by a barometer, are constantly fluctuating. With every such change the amount of the refraction changes. As the air grows warmer, its refractive power decreases, as it grows denser, the refraction increases. For the accurate work of the astronomer, refraction is extremely troublesome; for the more

PLATE II

Distortion of the Sun's Disc by Refraction

or less rough work of the navigator tables have been prepared, as the result of many experiments, which give the amount of refraction with sufficient accuracy.

Among the noticeable and striking effects due to refraction are the so-called mirages. On a perfectly calm day when there is no breeze to stir up and thoroughly mix the air, the atmosphere will arrange itself in horizontal layers of different densities over the surface of the ocean or of a desert. The rays of light from distant land or from a ship will travel in a curved path, and the land or ship will appear raised far above the horizon; at times even appearing inverted.

Dip of the Horizon

The fact that the earth is a sphere of some size and that observations must be made from its surface introduces complications into all astronomical measures. For the navigator the most important of these is the so-called "dip of the horizon."

At sea the altitude of the sun or of a star is always measured from the visible horizon. Now as the eye of the observer is and must be elevated above the surface of the water, this visible horizon will always be below the true, or celestial horizon. The true horizon, it will be remembered, is the great circle traced on the heavens by a plane tangent to the surface of the earth at the place

of observation; it is everywhere 90° from the zenith. As the observer rises higher and higher above the surface, the visible horizon is depressed more and more: the visible horizon is more than 90° from the zenith, and from the masthead of a ship at sea one can see considerably more than one half of the heavens. This angular depression of the visible horizon is called the "dip," and its essential elements are shown in the following figure.

In this diagram C represents the centre of the earth, and O the eye of the observer at an elevaion PO above the earth's surface. OH, PH′, and CH″ are sections of the true horizon, for these lines, being parallel, all meet the celestial sphere in the same great circle, the celestial horizon. OT is a section of the visible horizon, for the observer can see any object on the surface between P and J and can observe any heavenly body which is above the line OT. The dip is the angle HOT.

Now the figure makes it perfectly clear that the altitude of the sun or a star measured from the visible horizon is always greater than when measured from the true horizon. For any given point on the earth's surface the true horizon is a fixed definite circle, while the visible horizon, on the other hand, is variable, depending upon the height of the observer's eye, and upon the condition and clearness of the atmosphere. In all astronomical problems of finding time, latitude, and longitude, the true horizon only can be used. On land, by

means of levels or plumb-lines, the position of the true horizon can always be found and the altitude of any body above it readily measured. But at

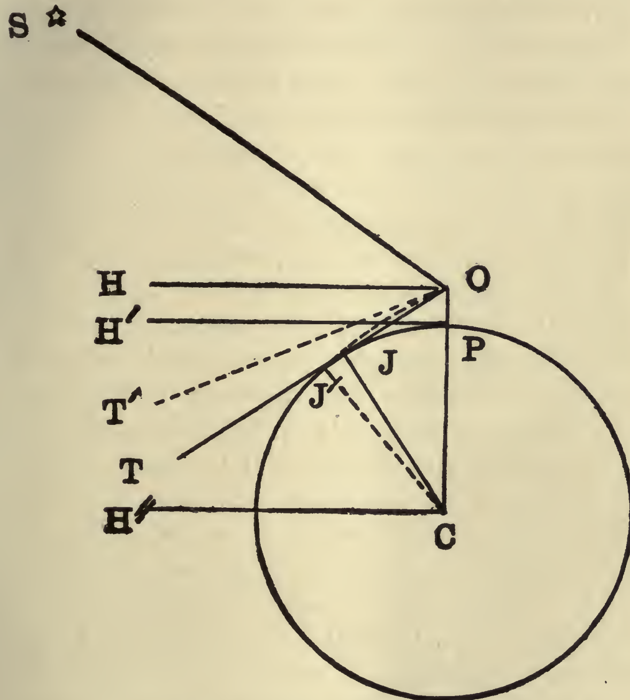

Fig. 2. Dip of the Horizon

sea this is impossible; the motion of the vessel prevents the use of artificial aids and the visible horizon only can be used. Hence every altitude measured at sea is too large and must be reduced by the amount of the dip.

The dip depends upon the size of the earth and upon the height of the observer above sea-level. The problem of finding the dip is a simple one in geometry and a very simple and accurate formula may be obtained quite readily. But such a simple formula would not be entirely satisfactory in practice, for, as has already been seen, atmospheric refraction bends the rays of light which come to our eyes and elevates the visible horizon. This is shown in the diagram; the rays which start from J′ travel a curved path and reach the observer's eye at O, and the point J′, which is now on the visible horizon, is seen in the direction of OT′. Thus the actual dip is generally not so great as the simple geometrical formula would lead one to believe it should be, and further it is evident that the actual sea horizon is usually farther from the observer than it would be if the earth had no atmosphere.

The amount that refraction raises the sea horizon is rather uncertain; it depends upon the state of the atmosphere. If along the path of the ray between J′ and O the air is of uniform density, then this path will be a straight line and refraction will have no effect upon the dip. This, however, is very unusual, for the density of the air depends upon its temperature, and the different strata of air are seldom at the same temperature. If we assume that the temperature of the lowest stratum of air is the same as that of the water, then when the water is cooler than the air at the observer's

station, the air will be denser at J′ than at O, and the ray of light will be bent as shown in the diagram. In this case refraction diminishes the dip. On the other hand, if the water is warmer than the air, then the atmosphere at J′ will be less dense than that at O, and the ray will be bent downward. In this rather unusual case the refraction will increase the dip, and diminish distance at which can be seen objects on the surface of the water.

The actual effect of refraction upon the dip can thus be determined by noting the temperature of the water and that of the air at the height of the eye. In fact tables can be computed which will show the amounts by which the dip must be corrected for various combinations of different temperatures of the air and water. Such tables are of value in unusual conditions, but for the ordinary purposes of navigation an average state of the atmosphere may be presupposed and the effect of refraction upon the dip computed upon this assumption.

The following simple expression gives the value of the dip in seconds of arc, allowing for the average refraction, when the height of the observer is expressed in feet.

$$D = 58''.82 \sqrt{h} \text{ (in feet).}$$

A very good value of the dip can aways be found by taking the square root of the number of feet of the observer's eye above sea-level, and calling the result the dip in minutes of arc. For an observer

9 feet above the water the dip will be 3′, for an observer 100 feet above water it will be 10′. Slight changes in the height of the observer make greater changes in the dip when the observer is near the sea-level, than when he is at quite a height above it. At an average height of 10 feet above the water, a change of 6 feet will alter the dip by one minute, while at a height of 100 feet, it requires a change of 21 feet to increase the dip by the same amount.

Hence when observations are made from a low deck, the height of the eye above the water must be accurately known. The rise and fall of the vessel on the seas will appreciably affect the dip. In clear weather it is thus better to take observations from the highest point that is convenient. From a high point of view the horizon is sharper and variations in the dip due to the ship's motions are less, than when the observer is close to the water.

On the other hand, when the water is calm and the atmosphere hazy, the observations should be made from as low a point as possible; for the haze may be so thick as to conceal the horizon corresponding to fifteen or twenty feet altitude, while showing clearly the horizon for a height of only a few feet.

Parallax

Every navigator is familiar with effects of "par-

allax," although the name itself may be strange and awe-inspiring. Whenever the distance of a ship from a rock or lighthouse is found by the familiar "bow and beam" or "four point" bearing, the principles of parallax are unconsciously used. Just as the bearing of the lighthouse changes with the motion of the ship, so the apparent position of the sun or of a planet in the heavens shifts as the observer passes from place to place on the surface of the earth. Two observers at different places on the earth's surface see the sun, at the same instant, in different parts of the heavens. But as the distance of the sun is very great as compared to the dimensions of the earth, this apparent shift in position, the parallax, is very small.

In predicting the position of the sun for noon of each and every day of the year, the *Nautical Almanac* can do so only for an observer in some standard locality. Now by common consent this standard locality, to which the positions of all bodies are referred, is one which no observer has ever yet reached—the centre of the earth. In the technical sense used in astronomy the parallax of the sun is the difference in direction in which it would be seen by an actual observer on the surface of the earth, and by an imaginary observer at the centre. If the sun were directly overhead, at the zenith, then the parallax would be zero, for the sun, the real, and the imaginary observers would all be in a straight line, and the

two observers would see the sun, therefore, in the same direction. If, however, the sun appear in any other part of the heavens than the zenith, it will appear to the real observer lower in the heavens than to the imaginary observer at the centre, or lower than as predicted by the almanac. The difference between the geocentric, or almanac, place and the actual observed place is the parallax. This parallax must always be added, therefore, to the observed altitude in order to obtain the true geocentric, or almanac, altitude.

The exact amount of this parallax depends upon two things: the distance of the body from the earth, and the altitude of the body above the horizon. It changes, therefore, from hour to hour as the body rises in the heavens, being at its greatest when the body is just rising above the horizon, and at its smallest when the body is on the meridian. Its greatest daily value is called the "horizontal parallax," and this horizontal parallax changes with the varying distance of the body from the earth.

In the case of the fixed stars the parallax is absolutely insensible; in the case of the sun it is never more than 9″, a quantity almost inappreciable in practical navigation. From the minuteness of this quantity, Captain Lecky, the distinguished writer on navigational subjects, deduces the practical result that parallax may be "left out in the cold" without detriment to navigation. For all observations of the sun this is probably true,

but when Mars is used the parallax may be considerably greater and it might be wise to apply the correction in such cases.

The *Ephemeris* or *Nautical Almanac* gives the value of the horizontal parallax of the different planets for different days of the year, and from these the "parallax in altitude" can readily be found. Simple tables are computed, which give, for different altitudes, the amount of the parallax which corresponds to different values of the horizontal parallax.

Conclusion

Every altitude measured at sea requires, thus, three corrections before it can be used in any computation for position; namely, corrections for dip, refraction, and parallax. Dip and refraction both apparently raise the body, they make it appear higher in the heavens than it really is, and their effects must be subtracted from the measured altitude. Parallax depresses the body, makes it appear lower than it really is, and its effect must, therefore, be added to the measured altitude.

NOTES AND PRACTICAL APPLICATIONS

1. According to Clarke, the following are the principal dimensions of the earth:

	feet		meters		miles
Equatorial radius,	20,926,202	=	3,678,249	=	3,963·296
Polar radius,	20,854,895	=	6,356,515	=	3,949·790
Difference,	71,307	=	21,734	=	13·506

Polar compression $= \frac{1}{293.46}$
Average density $= 5.527$
(*Water* = 1.)
Mass of the earth $= 6 \times 10^{21}$ tons

2. Formulas for dip and parallax:

(*a*) Excluding the effect of refraction the dip is given by:

Dip (in minutes) $= 1'.06\sqrt{h}$ (in feet).

(*b*) The correction to the dip in seconds for the effect of refraction is given by:

$$\text{Correction} = \frac{400}{D}(t_0 - t),$$

where,

D = uncorrected dip in minutes.
t_0 = temperature of the water in degrees Fahrenheit.
t = temperature of the air at height of the eye.

Hence:

When the air is *cooler* than the water, the correction is positive (+) and the dip *increased.*

When the air is *warmer* than the water, the correction is negative (–) and the dip is *decreased.*

(*c*) Including the mean refraction (average state of the atmosphere) the dip is given by:

Dip (in minutes) $= 0'.98 \sqrt{h}$ (in feet).

Distance (in miles) $= 1.16 \sqrt{h}$ (in feet).

(*d*) The parallax in altitude, p, of any body is given by:

$p = \pi \cos h$,

where:

$h =$ the observed altitude.

$\pi =$ the horizontal parallax as given in the *Nautical Almanac.*

3. Practical examples.

The examples in this and following chapters were taken from actual practice at sea. They show the methods of using the tables and correcting the observed altitudes. The dates of the various problems, however, have all been brought up to 1908, so that a single almanac will furnish the data for any one who wishes to check the figures. The Tables for dip, refraction, etc., will be found in the appendix.

(*a*) *Altitude of the Sun:*

At sea Nov. 26th, in D. R. latitude 40° 30′ N. the measured altitude of the sun's lower limb was 28° 32′; index correction, + 1′ 30″; height of eye, 30 feet. Required the sun's true altitude.

Observed altitude $\underline{\odot}$	=	28° 32′ 00″
Index correction	+	1′ 30″
Dip. (Table II)	–	5′ 23″
Refraction, etc. (Table IV)	+	14′ 21″
Correction for date (Table V)	+	11″
$\odot$'s true altitude,	=	28° 42′ 39″

This result differs by only 4″ from that obtained by using the regular tables for refraction and parallax, and the semi-diameter from the *Nautical Almanac*. As a matter of fact the "correction for date" from Table V can in general be omitted without sensible error. It is included in this and the following problems for the sake of completeness.

At sea, June 21st, the measured altitude of the sun's lower limb was 40° 6′; index correction, +1′ 0″; height of eye, 20 feet. Required the sun's true altitude.

Observed altitude $\underline{\odot}$	=	40° 6′ 00″
Index correction	+	1′ 00″
Dip (Table II)	–	4′ 24″
Refraction, etc. (Table IV)	+	14′ 57″
Correction for date (Table V)	–	16″
$\odot$'s true altitude	=	40° 17′ 17″

And this again differs by 4″ only from what would have been obtained by an accurate reduction with the *Nautical Almanac* and the tables of refraction and parallax.

(*b*) *Altitude of Star or Planet:*

On June 19th, the observed altitude of Vega was 41° 45′ 20″; no index correction: height of eye 15 feet. Required the true altitude.

Observed altitude	=	41° 45′ 20″
Dip (Table II)	−	3′ 49″
Refraction (Table IV)	−	1′ 5″
*'s true altitude	=	41° 40′ 26″

On Sept. 9th, the altitude of Saturn was measured as 23° 45′ 00″; no index correction: height of eye 15 feet. Required the true altitude.

Observed altitude	=	23° 45′ 00″
Dip (Table II)	−	3′ 49″
Refraction (Table IV)	−	2′ 13″
*'s true altitude	=	23° 38′ 58″

The parallax is so small that it can be neglected. Only in exceptional cases need the parallax of a planet be taken into account. When Mars is in opposition the parallax might amount to 10″ or more.

CHAPTER II

THE MOTIONS OF THE EARTH

The Rotation of the Earth

DURING the ages of Greek supremacy in thought and for many centuries thereafter, the earth was considered as immovably fixed at the centre of the universe. Philolaus, a Greek philosopher, however, who lived in the fifth century before the Christian Era, regarded the earth, as well as the sun, moon, and planets, as revolving about a great central fire, the earth turning about an axis as it revolved, so that this central fire should ever remain concealed from the inhabitants. From this time on many of the ancient writers seem to have favourably considered the idea of a rotating earth, but no one grasped the essential facts. The regular alternation of day and night was thought to be caused by an actual daily motion of the sun around the earth; the fact that the change from daylight to darkness and from darkness to daylight is caused by a simple rotation of the earth was not clearly recognised and did not become an accepted scientific belief until

some fifty years before Columbus discovered America.

To-day there are numerous ways in which the rotation of the earth can be demonstrated. For example, if a number of bodies be dropped from a very great height there is found to be a tendency for them to fall to the eastward. Small and perfectly round balls have been dropped with great care into deep mines, and careful measurements show that they strike the bottom a small fraction of a foot to the eastward of the point vertically under the starting-point. The top of the mine is farther from the centre of the earth than the bottom and is therefore moving faster toward the east, and a body dropped from the upper part retains its eastward motion as it falls and strikes to the east when it reaches the bottom. Similarly a ball thrown from a moving train partakes of the motion of the train and reaches the ground many feet in advance of the object at which it is aimed. As a result of many trials with bodies having a free fall of 520 feet, an eastward deviation of 1.12 inches was observed. According to theory this deviation should have been a trifle less, or only 1.08 inches.

But the best and most striking experimental proof of the rotation of the earth is that devised in 1851 by Foucault, who in that year swung his famous pendulum from the dome of the Pantheon. This pendulum consisted merely of a heavy iron ball supported by a thin flexible wire, which was

securely fastened to the dome in such a manner that the bob could swing freely in any direction.

Now if such a pendulum be started swinging, it will, unless disturbed, continue to swing in the same plane. If hung directly over the north pole and started swinging in such a manner that the bob moves in a line directly to and from the sun, it will continue so to swing and the bob will always move in a line directed towards the sun. But the earth rotates from west to east and the sun apparently moves toward the west, and, as the bob of the pendulum keeps pace with the sun, the direction of its swing would apparently shift around toward the west. In other words, the earth would turn around under the pendulum, and complete a revolution every twenty-four hours.

When such a pendulum is set up in any place other than the north or south poles, the effect will be similar but not identical. In the northern hemisphere the pendulum would appear to deviate slowly towards the right, in the southern hemisphere toward the left, and the amount of the deviation depends upon the latitude of the place in which the pendulum is supported. The reason for this deviation will be made clear by a moment's consideration. A point of the earth's surface at the equator moves eastward at the rate of one thousand miles an hour; from the equator this speed gradually diminishes from a thousand miles per hour to nothing at the poles, for at the poles

Plate III

Foucault's Pendulum at Columbia University

themselves the surface is stationary. In the latitude of New York this eastward motion of the rotating earth is somewhat greater than seven hundred and fifty miles per hour. When hanging at rest all parts of the pendulum partake of this eastward motion, and starting the ball swinging in a north and south plane will not destroy or alter this motion. When, however, the ball reaches the north end of its swing, it will be farther from the equator than when at rest, and it will then be travelling to the east a little faster than the point of the earth then directly underneath it. Hence in swinging to the north the ball will apparently deviate a little to the east of a north and south line. When the ball is at the south end of its swing, it will be moving a trifle slower than the earth underneath, and it will be left behind or deviate to the west of the south direction. At each swing of the pendulum, it will deviate more and more from a north and south line, and this apparent turning to the right will continue so long as the pendulum can be kept vibrating. In New York this shift is a little less than ten degrees per hour, an angle through which the hour hand of a watch moves in about nineteen minutes. This is sufficiently rapid for ordinary observation and thus such a pendulum makes the rotation of the earth clearly visible.

To the rotation of the earth combined with its ellipsoidal shape is due a phenomenon which has been known for over twenty centuries. The

"vernal equinox," the Greenwich of the heavens, is the point of intersection of the celestial equator and the ecliptic, the apparent yearly path of the sun through the heavens. Now, one hundred and twenty years before the Christian Era, Hipparchus found that this point is moving slowly to the westward along the ecliptic, advancing to meet the sun as each year it returns to the equator. Hipparchus called this motion the "precession of the equinoxes."

This slipping of the equinox backwards along the ecliptic is caused by a motion of the north celestial pole, which travels around and around a circular path among the stars. At present this motion is carrying the north pole towards the North Star, so that to future generations the North Star will be a more accurate guide than it is at the present moment. Very slowly, indeed, does the pole travel along this path: in the span of an ordinary lifetime it moves over a portion of the sky somewhat less than half the diameter of the full moon; it requires nearly 26,000 years for the pole to make one complete circuit of the heavens. Nor is this motion exactly uniform, at times the pole moves at a greater speed, and at times more slowly than the average, and it also wabbles a little to one side or the other of its prescribed path. The average motion of the pole is technically the "precession," while all the variations in this average motion, all the wabblings, are technically called "nutations"

Precession and nutation may be illustrated with a spinning top. When the axis about which the top spins is exactly vertical, the top apparently remains at rest or "sleeps," the axis pointing directly towards the zenith. When, however, the axis of the top is tilted, then the top itself begins to wabble; the iron point remains at rest on the floor, but the upper part of the top swings round and round in a circle which gradually increases in size as the top slows down. As soon as the spinning ceases the top falls over on its side. Thus the circular motion of what might be called the north pole of the top is caused by a combination of the rapid spinning of the top about its axis and of the force of gravitation which tends to make it fall over on its side.

In the case of the spinning earth the attraction of the sun and moon upon the protuberant matter near the equator replaces the force of gravitation which causes the top to fall over. If the earth were spherical or if the sun and moon were always in the plane of the equator, then there would be none of this tilting effect and the axis of the earth would, like the axis of a "sleeping" top, remain for ever pointing in the same direction.

This motion of the pole and the consequent precession of the equinoxes introduces complications into astronomical measurements. The positions of all the stars are apparently changing, for these positions are all referred to the vernal equinox, and the vernal equinox is itself in

motion. As this motion is accurately known, however, the apparent shift in position of any star can be calculated. These shifts have been calculated for the principal stars, and their positions at various dates are tabulated in the *Nautical Almanac.*

A second peculiar and at first sight startling effect of the rotation of the earth has been discovered within the last few years. The latitude of a place is not constant: the distance from the equator to any other point on the earth's surface is changing from day to day and from year to year. This variation of latitude was first definitely shown to exist by Chandler in 1891. It arises from the fact that the axis about which the earth rotates is not fixed in the earth; the north pole, or point where this axis cuts the surface, wanders around in an irregular curve, covering in its wanderings an area equal to nearly two city lots. As the equator is an imaginary circle everywhere 90° distant from the poles, it must oscillate back and forth over the surface, keeping pace with the movement of the pole, and thus changing the latitude of every spot on the earth's surface.

This variation of latitude is rather minute, the extreme shift being some 0″.6, which corresponds to an actual motion of 60 feet. That is, at one time each building in New York City is 60 feet nearer the equator than at other times. During the years 1893–1900 an extensive series of latitude

observations was made at Columbia University. The latitude was the smallest on September 15, 1895, and greatest on August 22, 1897, the total

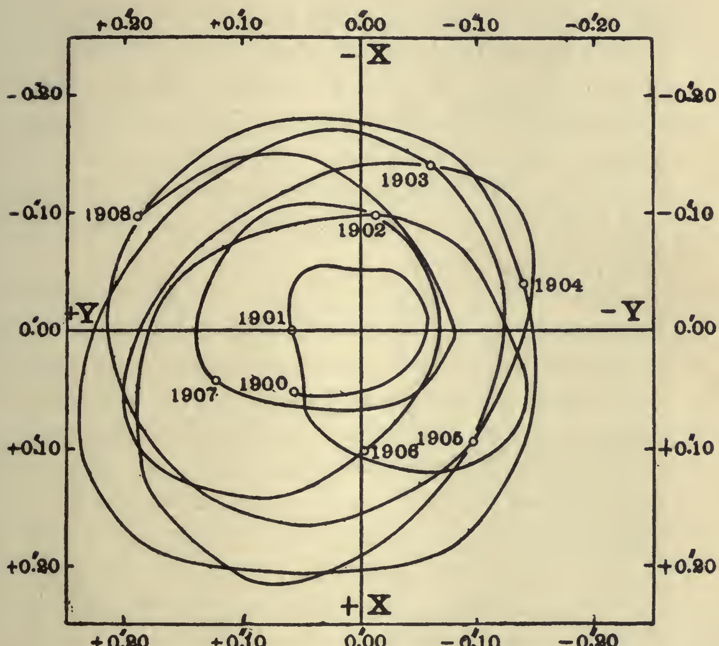

Fig. 3. The Wanderings of the Earth's Pole during the Years 1900–1908

variation between these extremes being 0″.696, or very nearly 70 feet.

Motion about the Sun

Besides the rotation about its axis, the earth

has an actual motion through space: it is travelling about the sun in an immense elliptic path, or orbit, as it is called. To us on the flying earth, however, it is the sun, not the earth, which appears to move, just as when we are on a smoothly running train the trees and hills appear to rush by the windows. In fact, for many centuries this apparent motion of the sun was considered as an actual motion of that body around the earth, and it was not until the time of Copernicus and Galileo that the motion of the earth was distinctly recognised.

This apparent motion of the sun through the heavens can be detected by the simplest observations. In general it rises each day in the east and sets in the west; but only on two days a year, March 21st and September 21st, does it rise exactly in the east and set exactly in the west. During the summer months the sun rises to the north of east and sets far to the north of west: in winter it rises to the south of east and sets to the south of west. Further, in summer it rises far higher than it does in winter; at noon in the latter part of June at New York the sun is nearly 70° above our horizon, in December it rises but a scant 25° above the southern horizon. The sun thus appears to oscillate back and forth, in summer being far to the north and in winter far to the south of the celestial equator.

If the stars were visible in the daytime it would be seen that the sun also moves slowly and steadily

to the eastward. While the stars cannot be seen in the day and the motion of the sun among them actually observed, yet the effect of this motion can be easily noted by watching the constellations at night. At midnight that portion of the heavens which is directly opposite the sun will be on the meridian. At twelve o'clock on a clear December night, the beautiful constellation of Orion will be found on the meridian about half-way between the southern horizon and the zenith. Each succeeding night Orion will reach the meridian about four minutes earlier, and after a week has elapsed it will be found on the meridian at half-past eleven instead of at twelve. By the middle of February Orion culminates in the early evening and by spring it is visible in the western sky for a few minutes after sunset. The constellation thus appears to move westward towards the sun, and as each and every other constellation partakes of this westward motion, it is clear that this apparent westward motion of the stars is in reality the effect of the eastward motion of the sun.

By combining this eastward motion of the sun among the stars with its north and south oscillation it is easily seen that the actual path of the sun around the heavens is in a path inclined $23\frac{1}{2}°$ to the equator. This path is called the ecliptic and in it the sun makes one complete circuit of the heavens in a year. It must always be remembered, however, that this apparent motion of the sun is due to an actual motion of the earth in the

opposite direction; the apparent eastward motion of the sun indicating a real motion of the earth to the westward.

Now, while it is customary to speak of the period in which the earth travels about the sun as a year, yet a little consideration will show that the term year is not definite, unless certain careful distinctions are drawn. In fact there are several different kinds of years, depending upon the point in the orbit from which the period is reckoned. A "sidereal year," for example, is the interval between two successive returns of the sun to the same position among the stars; a "tropical year," the interval between two successive returns of the sun to the vernal equinox. This latter year is that upon which the seasons depend and is the year of ordinary, every-day conversation. These two years, the sidereal and the tropical, are not of the same length, for, as we have seen, the vernal equinox is in motion, is moving along the ecliptic in the opposite direction to that in which the sun appears to move. In fact it was this difference in the lengths of the years which led Hipparchus to the discovery of precession.

The sidereal year measures the actual time that it takes the earth to make one complete revolution about the sun. It is 365.25634 days, or 365^d 6^h 9^m and $9^s.4$ long. The tropical year is a trifle shorter, its length being 365^d 5^h 48^m and $47^s.5$. Further the actual tropical year is not of constant

length, for the vernal equinox does not move forward uniformly. When the equinox is moving faster than usual the year will be a few seconds shorter, when the equinox moves more slowly, the year will be longer. But as this variation in the motion of the equinox, or nutation, as it is called, is very small, the actual difference in the lengths of various tropical years will be very slight. The average value is that given above.

Now the great orbit or path in which the earth moves about the sun is some 185,000,000 miles in diameter. This path, however, is not circular; it is a sort of oval curve, a curve known to mathematicians as an ellipse. And further the sun is not at the centre of the curve, but slightly nearer one end, at a point called the focus. Different parts of the curve are thus at different distances from the sun, and as the earth travels around and around its orbit each year, it will be at continually varying distances from the sun. In January the earth is nearest the sun, or is at perihelion, and at this time it is some 3,000,000 miles nearer than in midsummer. The apparent size of a body varies inversely with its distance; the farther away a body is, the smaller it appears. The sun, therefore, should appear larger in January than in July, and careful measurements show this to be so. On January 3d the apparent semi-diameter of the sun is some 32″ greater than on July 4th.

Further, the earth does not move at a constant speed in this orbit; when it is nearer the sun it

moves faster than when at a greater distance. On the average the earth moves forward at a rate of about nineteen miles per second, about fifty times as fast as the bullet from a modern rifle. At perihelion this speed is increased to 19.3 miles per second, while at aphelion it drops down to only 18.7 miles. A direct statement of the speed at which the earth moves at various distances from the sun involves complicated mathematical formulas, but there is an indirect relation between speed and distance, which was discovered by Kepler nearly three hundred years ago. This indirect statement is involved in what is known as Kepler's second law of planetary motion. His first two laws which apply to the earth and all the other planets of the Solar System are so important and so easily understood that they are here reproduced. They are:

1. Each planet describes about the sun an ellipse, the sun being at one focus.

2. The straight line joining a planet to the sun sweeps over equal areas in equal intervals of time.

The first law gives the shape of the path that each planet describes about the sun: the second, the speed with which the planet moves in various parts of its orbit. In order that the areas swept over each day by the line joining the planet to the sun shall always be equal, the planet must move faster when nearer the sun than at other times. Now these two laws are exemplified in the

following diagram, which, however, does not represent the real path of any definite planet: the eccentricity of the ellipse being much exaggerated. For the actual orbits depart so slightly from circles, that, in a diagram drawn to scale, the eye

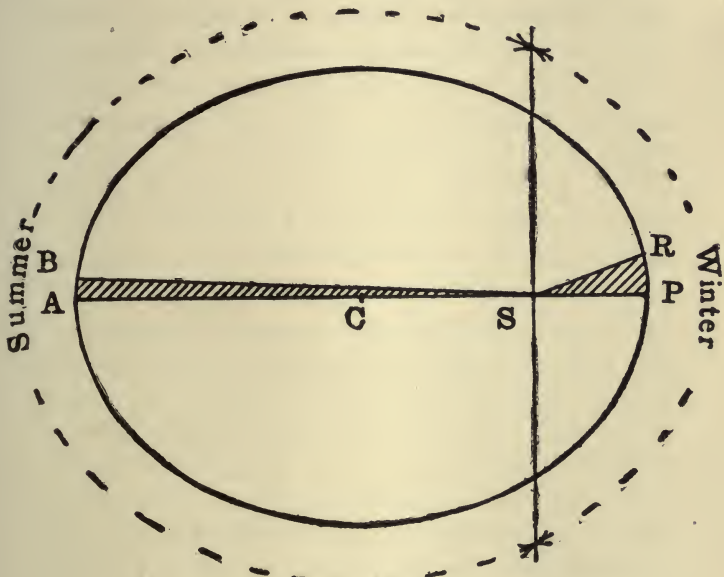

Fig. 4. The Orbit of the Earth

could hardly distinguish the difference between ellipse and circle.

The half major axis of the ellipse, or the distance CA, is called the "mean distance," and in the case of the earth this is about 93,000,000 miles. This distance of the sun from the earth is so great

that the mind fails to grasp it unless some concrete illustration is used. A seasoned walker can average 5 miles per hour and if he walk 20 hours each day he will average 100 miles per day. Walking thus day after day and year after year such a pedestrian would require nearly twenty-six centuries to cover the distance between here and the sun. Again another illustration. The great turbine ship *Mauretania*, the largest and fastest of the great ocean liners, made 624 nautical miles between noon of one day and noon of the next. This is at the rate of nearly 29 ordinary, or land, miles per hour. Now if the *Mauretania* travelled at that average speed day and night, without a stop, she would require nearly four centuries to pass over a distance equal to that of the sun from the earth. Had Columbus sailed from Spain in the *Mauretania* and voyaged toward the sun he would but now have reached his destination.

The sun is at one focus, S, and the distance C S between S and the centre determines the shape of the curve. The ratio of CS to CA is called the "eccentricity" of the orbit, and this eccentricity differs widely for the various planets. In the case of the earth it is about $\frac{1}{60}$; the focus being about 1,500,000 miles from the centre of the orbit. The point at which the planet approaches the sun most closely is called the "perihelion"; the point at which it attains its greatest distance, "aphelion." In this elliptic orbit the earth travels about the sun from west to east, moving at every point at such a

speed that the law of areas holds true. When at perihelion the earth will in one week pass from P to R, when at aphelion, a somewhat less distance from A to B, in the same interval. The areas of the sectors PSR and ASB are equal, and in the diagram these equal areas are shaded. The earth passes through perihelion on or about the first of the year, and, thus being nearer the sun in winter, it passes over one half, or 180°, of its orbit faster than it does in summer. In fact the northern winter is about three days shorter than the summer.

As a whole the earth receives nearly six per cent. more heat in twenty-four hours in December, than it does in a corresponding interval in June. For the amount of heat received increases as the square of the distance from the sun decreases, and in December the earth is about three per cent. nearer the sun than in June. But December is midsummer in the southern hemisphere and hence the southern summer should be hotter than the northern. On the other hand the southern summer is three days shorter than the northern and this shortness of the summer about counterbalances the extra amount of heat received each day, so on the whole there is no radical difference between the summers in the two hemispheres.

Far different, however, is the winter climate in the two hemispheres: the southern winter is both longer and colder than the northern. This applies, of course, only to the hemispheres as wholes. The

climate of any particular place is so modified by local conditions that it is next to impossible to make definite comparisons of two widely separated points and to determine that one actually receives less heat than the other. But if the two hemispheres were identical as to land, water, elevations, forests, etc., then it is clear that of two places similarly situated on opposite sides of the equator the southern one would have the colder winter, and the hottest day in summer.

NOTES AND PRACTICAL APPLICATIONS

1. *Astronomical Constants:*

Parallax of sun	$8''.80$
General precession (1900)	$50''.2564$
Constant of nutation	$9''.21$
Constant of aberration	$20''.47$
Time required for light to travel from sun to earth	$498^s.5$

2. *Earth's Orbit:*

Longitude of perihelion (1900)	$101^\circ\ 13'\ 15''.0$
Obliquity of ecliptic (1900)	$23^\circ\ 27'\ 8''.26$
Mean daily motion in orbit	$0^\circ\ 59'\ 8''.1928$
Eccentricity of orbit	0.01675

Mean distance of earth from sun,	92,897,000 miles
Greatest distance " " "	94,458,000 "
Least distance of " " "	91,336,000 "

Length of Julian year, 365.25 days $= 365^d$

Length of Tropical year, 365.2422 days $= 365^d\ 5^h\ 48^m\ 45^s.51$

Length of Sidereal year, 365.2564 days $= 365^d\ 6^h\ 9^m\ 8^s.97$

CHAPTER III

THE SUN

THE heavenly bodies may be divided into two classes: the so-called "fixed stars" and the bodies of the solar system. The fixed stars are countless in number. They are seen always in the same portions of the heavens and appear in the most powerful of telescopes as mere points of light without form, size, or shape: the bodies of the solar system on the other hand are few in number, they wander through the heavens visiting the different constellations, and by the aid of the telescope their shape, size, and surface conditions may be studied. Of all these celestial bodies—stars, sun, moon, and planets—the most important to the inhabitants of the earth is the sun.

The countless myriads of stars and the numerous planets could be blotted out of existence without sensibly affecting our daily life; the moon might be shattered into fragments and dispersed throughout space without materially changing the conditions under which we live and exist; the nights

would be dark, the tides and currents which sweep our coasts would be radically modified, and the lengths of the day and the year might even be changed to an appreciable amount, but we could still go on living our lives, pursuing our business and our pleasures as we do to-day. But if the sun ceased to shine the days of the world would be numbered.

The sun is the centre from which is derived the heat, the energy, the life of the earth. In winter the sun does not rise so far, nor remain so long above our horizon as in summer, and to the differing amounts of heat thus given us are ascribed our ever-varying seasons. The variations in climate, the difference between the torrid heat of the tropics and the rigours of an arctic winter, are caused by the radically different amounts of solar heat received. A sensible increase or diminution of the solar radiation would modify the climate of the entire world. A radical decrease in the amount of heat received from the sun would cause the polar ice to spread toward the equator, would produce an age of ice and snow and bring death and destruction to the inhabitants of our world. The earth, undoubtedly, has internal heat of its own, but if the sun ceased to warm the atmosphere, for even a single month, the earth would grow cold and uninhabitable.

From the earliest times the principal facts about the sun, those which are of special interest to the navigator, have been known. From

pre-historic ages the path of the sun through the heavens has been recognised and the length of the year known to within very narrow limits. One hundred and thirty years before the Christian era Hipparchus had determined the length of the year, with an error of less than seven minutes; three centuries before Christ, Aristarchus had measured the relative distances and sizes of the earth, and moon, and the sun. He knew the sun as a vast globe many times larger than the earth and some millions of miles distant.

During the twenty and more centuries which have elapsed since Aristarchus lived and made his crude measures, many wonderful inventions have been made, giant telescopes have been constructed, and powerful methods of mathematical analysis brought into use. Corresponding advances have been made in our knowledge of the size and distance of the sun. It is now known that the sun is a great globe between 860,000 and 870,000 miles in diameter and nearly 93,000,000 miles distant. The direct determination of this distance is impossible. From the laws of planetary motion, however, we know the correct shapes and the relative sizes of all the planetary orbits; we have a correct map of the solar system, but a map without a scale. If any distance on the map be found, the scale can be at once determined, and all other distances found. Now, at times, Mars approaches the earth much closer than the sun, and at these times the distance between the earth

and Mars may be found and the scale of the whole map determined. This distance is found by means of "parallax"—a very long name for a very simple thing. Every navigator is familiar with the "bow and beam" or "four-point bearing," by which a ship's distance from a rock or lighthouse may be found. A vessel sailing due west finds a light on her starboard bow bearing northwest; after running on her course for five miles by the patent log the light is abeam, or bears north; at this moment the light will be five miles distant, the distance run by the log and the distance of the vessel from the shore being exactly equal. Now this change in the apparent direction of the lighthouse, caused by a real change in the vessel's position, is what is called in astronomy, "parallax." And by means of this parallax or change in bearing, together with the distance travelled by the ship, the distance from the lighthouse can be found by simple geometry.

In astronomy the parallax of a body, Mars for example, is the difference in direction in which it is seen by an observer, or observers, in two different positions. And as with the ship, as soon as the parallax and the distance through which the observer has moved is known, then the distance of Mars can be found. But, while in the case of the vessel the change of bearing, or parallax, is four points, in astronomy the parallax of Mars is only 20″ or a trifle less than $\frac{1}{2000}$ of a point. The best direct measurement of this little shift was

that made by Sir David Gill on Ascension Island in 1877. The method he used was simple, effective, and easily understood. The planet must be observed from two or more different positions: now Gill utilised the daily rotation of the earth on its axis to carry him from point to point as he made his observations, just as the ship carries the navigator by the lighthouse as he makes his "bow and beam" bearing. In the early evening as the planet was just rising above the eastern horizon he observed carefully its direction, its bearing, as shown by its place among the stars. Six hours later the earth had made a quarter revolution on its axis and Gill had been carried nearly six thousand miles to the eastward. Again he measured the position of the planet in relation to the fixed stars, and the apparent shift or change of position among the stars since the earlier observation was the parallax. The two observations were never actually made just six hours apart: one was made in the early evening and the other just before sunrise the next morning; but by noting the time which elapsed between the two observations, the distance through which he had been carried by the rotation of the earth could be readily calculated, and this together with the corresponding shift of Mars enabled him to compute the distance of the planet.

The Island of Ascension was chosen for these observations because it is but 8° from the equator, and consequently the daily path of the observer

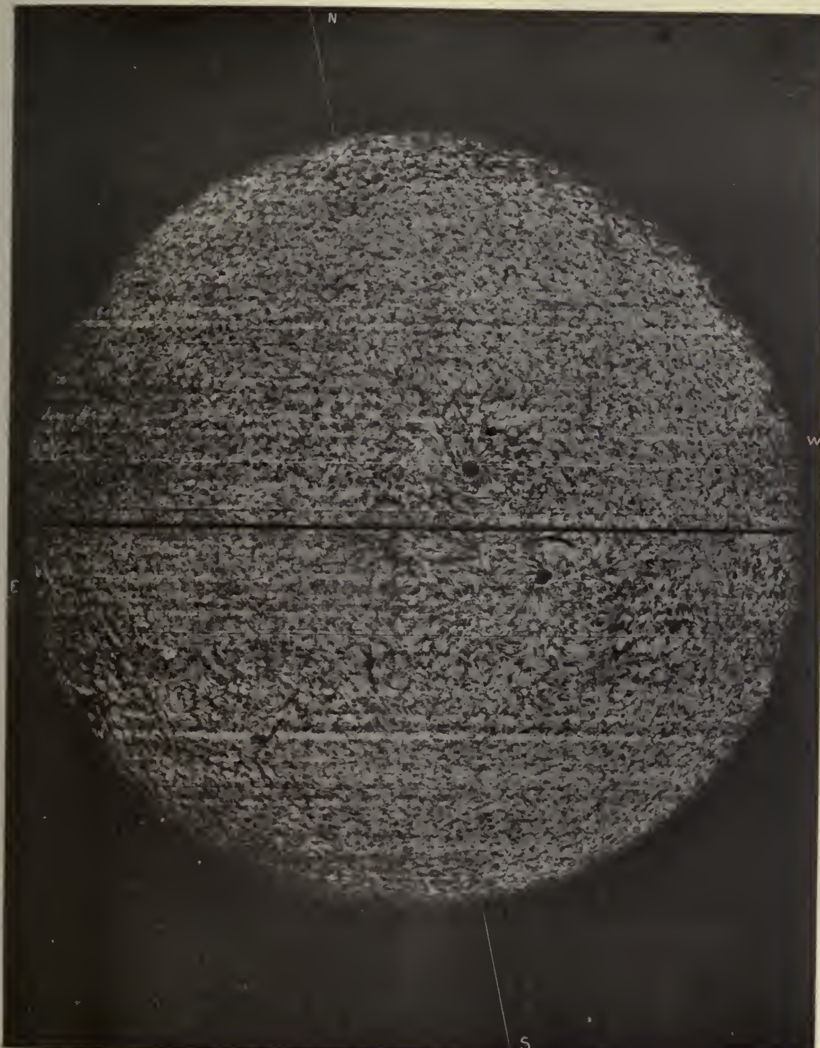

PLATE IV

Solar Cyclones, Photographed at the Carnegie Solar Observatory

is nearly the longest possible. At the pole an observer would be stationary and the method inapplicable. For the accurate measurement of the shift Gill took with him a heliometer, the most delicate instrument known to astronomy. With it he made over 350 determinations of the parallax of Mars, and these measurements are probably the most precise of modern astronomy; the probable error in the determination of the planet's position on any single evening being only about $\frac{1}{10}$ of a second of arc, or $\frac{1}{400,000}$ of a compass point.

The sun itself is a globe of incandescent gases and vapours and so nearly spherical in shape that the most accurate measures of modern times have failed to show any distinct departure from that form. Newcomb and Ambronn regard the heliometer measures as conclusive evidence that the sun is sensibly a sphere. Yet these measures are so difficult to carry out with accuracy, that too much weight should not be placed upon them. All the other bodies of the solar system, like the earth, show distinct elliptical discs, and it is barely possible that the sun may be ellipsoidal to a very minute extent. It is certain, however, that the extreme difference between the equatorial and polar diameters of the sun cannot exceed $0''.25$, or, in other words, that the equatorial diameter does not exceed the polar by so much as one hundred miles.

The great size of the sun as compared to the earth can best be shown by illustrations. If the

American fleet could have proceeded directly around the world, steaming day and night at its average seagoing speed, it would have completely circumnavigated the globe in one hundred days. Proceeding at the same rate it would take such a fleet thirty-one years to complete a voyage around the sun. Again, so vast is the sun, that were it a hollow globe, the earth and moon could be transported and placed within the hollow; the earth at the centre and the moon in her orbit, 250,000 miles from the earth, would never be but little more than half way out towards the sun's surface.

When the sun is viewed through a telescope its surface is often found to be covered with dark, irregular spots. And these dark sun-spots are seen to be in motion; when watched from day to day they appear to travel slowly across the luminous disc. They appear on the eastern edge of the sun, move slowly toward the centre, cross the disc, and disappear at the western limb. Whether the spots pass through the centre, or along a shorter chord above or below the centre, the actual time of crossing the disc is always about the same number of days. In this motion of the spots is recognised an actual rotation of the sun just as the earth rotates each day about its axis. In the case of the sun, however, the rotation is extremely slow and stately, each sun day being about twenty-six of our days in length. There is, however, a marked difference between the rotation of the sun and the earth: the sun does not

rotate as a whole. Spots near the equator complete an entire revolution in a much shorter period of time than do spots in high latitudes. This equatorial acceleration is somewhat more than two and a half days; a spot on the equator requiring not quite twenty-five days to complete a single circuit, while a spot in latitude 45° requires twenty-seven and one-half days. Spectroscopic investigations show that this peculiarity cannot be explained by a mere drift of the spots over the surface of the sun, for the entire surface participates in this rotation. The latest results obtained by Adams at the Mount Wilson Observatory indicate that particles at the equator of the sun rotate in 24.6 days and that the speed of rotation steadily decreases from the equator towards the pole. In latitude 50° the period of rotation is about 28.8 days, in latitude 60°, 31.3, while in latitude 80°, the particles composing the surface of the sun require nearly 35.5 days to complete one circuit.

This rotation is proof sufficient that the visible surface of the sun is not a solid rigid body; it must be liquid or gaseous. The earth's equator crosses the west coast of Africa a few miles north of the mouth of the Congo River, while far to the north and upon the same meridian of longitude is to be found Genoa in latitude 45°, and Cristiania, the capital of Norway, in latitude 60°. Now if the earth rotated as does the sun, then a day in Africa would be 24, a day in Italy 28, and a

day in Norway over 31 hours long. At the end of one Norwegian day, Genoa would have slipped 40° to the east and be where now is the Caspian Sea, Africa would have slipped 100° and have taken the place of Borneo. At the end of three and a half days Africa would have occupied all positions on the equator, and again be found directly south of Norway, but Italy would be lost in the deep waters of the Pacific to the eastward of Japan.

While the matter near the surface of the sun must thus be very tenuous, yet as a whole the sun is much more dense than any gas known on the earth. Its average density is nearly one and a half times that of water, about the same as the lighter rocks on the earth's surface. This average density is found from the known diameter of the sun and from the total amount of matter which it contains. This latter can be calculated from the force of attraction, which the sun exerts upon the earth and upon the other bodies of the solar system, and this force of attraction can be measured by the length of time required for each planet to travel about the sun; by the length of the year for example. It can readily be shown that if the amount of matter in the sun were quadrupled, then the year would be only one half as long as at present.

If the sun and earth were at the same distance from a given body then the sun would attract that body 330,000 times more powerfully than does

the earth. The sun contains 330,000 times as much matter as the earth. Yet this matter fills such a large globe (the sun's diameter is 110 times that of the earth) that the average density of the sun is only about one quarter that of the earth. And at the sun's surface, moreover, the attraction of gravity is only twenty-eight times that upon the surface of the earth. Here if a small body be dropped it falls 16 feet in the first second, on the sun such a body would fall 444 feet and at the end of the second second it would be moving with the speed of a cannon-ball. A man who here weighs 175 pounds would, if transported to the sun, weigh nearly two and a half tons, and would be crushed under his own weight.

The spectroscope shows that the sun is composed of very much the same kind of matter as is the earth. Most of the ordinary elements of which our earth and rocks are composed are found in the sun in the form of incandescent gases. Sodium, carbon, iron, lead, copper, silver, and many other terrestrial elements are as common on the sun as they are on the earth; in fact, so far as chemical constitution, the sun seems to be nothing more than an immensely large and intensely heated earth.

The effective temperature of the sun's visible surface is at least 10,000° Fahrenheit. Some idea of what this means may be gathered when it is remembered that lead melts at about 800°, cast iron at a little over 2000°, and that the tempera-

ture of the molten metal in a blast furnace is under 4000.° Of course this estimate of the sun's temperature is more or less approximate, for from the very nature of the problem it is impossible to reach an accurate conclusion. We cannot put a thermometer in the sun and directly measure its temperature. But we know that the character of the radiation emitted from a body depends upon its temperature. When a cannon-ball is heated, it first sends out rays which can be felt as heat, but which do not affect the eye. If placed in a dark room, the heat from the ball can be felt at some distance, but the ball itself cannot be seen. As the ball becomes hotter it turns a dull-red and becomes faintly visible. After a while, as its temperature rises, it becomes white hot. The kind of radiation given off by a body thus affords an indication as to its temperature, and we can study the character of the sun's light, and thus indirectly estimate its temperature.

Another similar problem relates to the amount of heat, or energy, given off by the sun and received by the earth. Heat and temperature are not merely different words for the same thing; they are essentially different, heat is a form of energy, while temperature is the amount of heat in each molecule of a body. Temperature represents relative concentration of heat; it is a measure of intensity, not of quantity. A body may contain very little heat and yet be at a high temperature. A pint of boiling water contains far more heat

than the flame of a candle, but the temperature of the flame is the higher. It takes twice as much heat to raise a quart of water to the boiling point as it does for a single pint; yet when both are boiling, the temperatures are the same. Quantity of heat may thus be measured in terms of the amount of water it will raise from one temperature to another, and the unit quantity is termed a "calorie." Thus temperatures are measured by degrees, quantities of heat by calories.

Now while the temperature of the sun is more or less a matter of inference and speculation, the quantity of heat emitted by it and received by the earth can be measured with considerable accuracy. The presence of the atmosphere introduces some difficulty, however; for the sun's rays must pass through many miles of atmosphere before they reach the measuring apparatus, and in this passage a large proportion of the energy is absorbed. Langley found that nearly 40% of the sun's energy thus failed to reach the surface of the earth, but it is not to be assumed that this absorbed energy is lost to the earth; on the contrary, it warms the atmosphere and helps towards making our globe habitable.

The latest researches show that the total quantity of heat and light received by the earth from the sun is tremendous. In each hour sufficient heat falls upon the upper layers of the earth's atmosphere to melt a sheet of ice nine tenths of an inch thick; in a year the heat from the sun would

melt a sheet of ice covering the entire surface of the earth and 164 feet thick. Another way of illustrating the immense amount of energy received from the sun is by means of the mechanical power it would produce if completely utilised. Upon every six and a half square feet of the earth's surface there is received on a cloudless day, when the sun is directly overhead, sufficient energy to develop one horse-power. An engine of a hundred horse-power will drive a small steamer, or furnish electricity enough to light a small village, and each city lot receives at high noon solar energy enough to run four such engines. Upon the deck of the *Lusitania* there is poured energy sufficient to develop five thousand horse-power. Unfortunately, however, this energy cannot be successfully utilised in running steamers and factories, for it is extremely variable. A passing cloud will interrupt the flow of solar energy, and in the morning or the afternoon, or upon stormy days, a very small part only of the solar energy actually reaches the earth's surface and becomes available as power. Some solar engines have been constructed, which run successfully during a few hours of a bright, clear day, but they have never been made a commercial success.

This heat energy which the sun is constantly pouring forth must come from some almost inexhaustible supply. It cannot come from combustion, for if the sun were solid coal and were burning in pure oxygen, it could not continue to supply

heat at the present rate for so long as five thousand years. In that period the sun would be entirely burnt up and would cease to exist. Yet we know that the sun has existed for many millions of years and during immeasurable ages has given forth heat and light without stint. The heat of the sun is derived from the motions of its constituent particles of matter. Heat is a mode of motion, and heat may be transformed into motion, or motion transformed into heat. When a moving body is stopped its energy of motion appears as heat and it makes no difference whether the stoppage be sudden or gradual, the total amount of heat produced is the same. A brick from the top of a tall building, a shooting star from the unknown wilds that surround us, each on striking the earth imparts a little heat. Every meteor, every body that falls into the sun, and an enormous number there must be, increases its store of heat. By its fall into the sun each particle of matter produces five thousand times as much heat as an equal weight of the best coal when burnt under the most favourable conditions. Were the earth to fall into the sun enough heat would be generated to last for ninety-five years; if the entire system of planets collapsed on the sun there would be liberated sufficient heat to supply the solar furnaces for over 46,000 years. Now the sun is gradually falling into itself, the outer layers are falling towards the centre; it is shrinking, growing smaller. And this contraction, this falling in

of the outer particles of the sun, produces the immense store of energy on which we are continually drawing. With the present size of the sun it is only necessary that its diameter be decreasing about 220 feet a year, four miles a century, in order that the supply of heat energy may continue the same year after year. This contraction is so slow as to be impossible to detect with any instrument now in use; after the lapse of many centuries the shrinkage will have proceeded far enough for us to measure.

The visible surface, or "photosphere," of the sun is not equally bright throughout. The central portion of the disc is the most brilliant, the edges being quite dark in comparison. This gradual decrease in brilliancy from the centre outward is due to an invisible atmosphere of permanent gases which surrounds the visible portion of the sun. This enveloping layer of vapours absorbs a portion of the light emitted by the hotter central photosphere and absorbs a greater proportion of the light from the edge of the disc, for the light from this portion of the sun must pass diagonally through this envelope and thus traverse a much longer path inside the absorbing vapours than the light which comes from the centre. Besides this darkening of the limb, the photosphere shows several other characteristic phenomena, among which the most striking are the dark spots, which have been mentioned in connection with the sun's rotation.

These spots appear either singly or in groups, and occasionally one is so large as to be easily visible to the unaided eye. Against the surrounding brilliant surface of the sun they appear a deep velvety black, but this colour is only relative. The darkest part of the blackest spot is in reality far more brilliant than the electric arc. These sun spots are short lived; they seldom last for more than three or four months, but during this short period they sometimes reach an enormous growth. Not infrequently a spot measures 20,000 miles in diameter, although the average is much less. In March 1905, a spot covered $\frac{1}{300}$ part of the sun's disc. It was clearly visible without the aid of a telescope and its area was nearly forty times that of the entire surface of the earth.

It is now quite well established that these spots are cavities, or local depressions in the solar surface. But it is not certain that the floor of this cavity is below the average level of the solar surface. Certain observations seem to indicate that in the neighbourhood of a spot the whole surface is raised and that the spot is a depression in this elevated portion, like a crater on the top of a low mountain. This crater-like formation of the spots offers a plausible explanation of their dark appearance. It is probable that in the neighbourhood of a spot eruptions take place and heated matter is ejected. The removal of the supporting material causes the surface to sink in and this sink is filled by the colder gases and

vapours of the solar atmosphere. Thus the light from the bottom of the well must pass through a much greater thickness of these cool vapours than the light from the surrounding surface, and in this passage a great proportion of the light is absorbed, and the spot is made relatively dark.

The spots are confined to certain well-defined zones of the sun's surface: few have been found at the equator and none have ever been observed nearer the poles than latitude 45.° Further they are periodic; their average number waxes and wanes in regular periods of eleven years. During a minimum practically no spots are visible, days and weeks often passing without a single spot marring the brilliant solar surface. Then a few small spots appear, and gradually the number and the size of the spots increase, until after a lapse of some five and a half years portions of the surface are constantly covered with large and small spots. Hardly a day passes, at the time of a maximum, without several spots being visible, and occasionally the spots are so numerous as to form two great belts around the sun, one on each side of the equator. This periodicity in the sun-spots is difficult to explain. It is probable, however, that the eleven-year cycle is a natural period, due to the physical condition of the sun as a rotating, cooling mass of gas; due to causes inherent in the sun itself and not to any outside influence.

While to the practical man the appearance of these spots on the sun may be regarded as a matter of little importance, yet it can be clearly shown that they have a direct influence on the earth; an influence which may materially affect the lives and well being of her inhabitants. The spots have a direct connection with magnetic and electric storms on the earth; when spots are numerous, magnetic storms are frequent, and the magnetic needle is in a constant state of agitation. Again they may affect the average temperature of the earth, and even have some influence on storms, rainfall, and the like. While, however, such influence may be suspected, it has not been conclusively proved, except in the case of electric and magnetic phenomena. It is certain that the central portion of a spot radiates less heat than the surrounding parts of the sun's surface, but whether the sun as a whole gives out less heat, when spotted, than it does at other times, is not known. The fact that spots exist on the sun shows that the activity of the sun is then greater, and the greater activity may overcome the lack of heat in the spots themselves. Newcomb made an exhaustive investigation and reached the conclusion that the world as a whole is a fraction of a degree cooler at those times when spots are less numerous on the sun, than at times of a sun-spot maximum.

While under ordinary circumstances, the spots are the most conspicuous features of the solar

5

surface, yet at times other striking phenomena are observed. This is especially the case during the few brief moments of a total solar eclipse. When the moon cuts off the last ray of direct sunlight, there instantaneously appears around the black disc of our satellite a brilliant, broad white, flickering halo, while close to the edge of the moon may be seen two or three bright red flames. The halo is the mysterious and elusive "corona," the red flames, the "prominences." By the aid of the spectroscope these prominences may now be seen and studied at any time, but the corona has never been seen except during a solar eclipse. The study of its nature is thus limited to a few moments (four or five minutes at the most) every few years. For this reason, whenever an eclipse occurs, expeditions are fitted out and sent to the most favourable locations, and the astronomer utilises every moment of totality in obtaining photographs and spectographs for measurement and study.

The prominences are eruptions from the layer of permanent vapours which surround the photosphere; their brilliant colour being due to the presence of hydrogen. These protuberances are of all sorts of fantastic shapes and some of them reach to an immense height. Young records a prominence which extended 350,000 miles beyond the edge of the sun. Great as this distance may seem, the corona extends even farther. This brilliant halo is made up of an intricate system of rays and streamers, which undergo periodic

PLATE V

Prominences on the Sun, August 14, 1907, Photographed at the Yerkes Observatory

changes in size and shape. The corona apparently consists of minute solid, liquid, and gaseous particles; matter ejected from the sun, meteoric matter, and minute dust-like planets. It is extremely tenuous, containing less matter per cubic inch than the best vacuum we can produce on the earth: the amount of matter in the corona is equivalent to a single dust particle in every fourteen cubic yards.

NOTES AND PRACTICAL APPLICATIONS

1. *Dimensions of the Sun:*

Mean apparent semi-diameter	16′ 0″.0
Greatest " "	16′ 16″.4
Least " "	15′ 44″.2
Mean semi-diameter	432,400 miles.

2. *Rotation of Sun:*

The periods of rotation of different parts of the sun are as follows:

Latitude	*Period, days*
0°	24.57
4	24.79
11	24.98
15	25.21
20	25.64
30	26.35
50	28.82
60	31.25
65	31.55
75	33.21
80	35.26

CHAPTER IV

THE STARS AND PLANETS

The Solar System

JUST as the earth revolves about the sun, so do countless other bodies—planets, planetoids, and comets. All these together form one great unique group, the solar system, of which the sun is absolute ruler, the czar. In the heavens might makes right and the strong rule the weak, and the sun rules over his system because of his vastly superior strength, or mass. The sun contains over three hundred thousand times as much matter as the earth, nearly one thousand times as much as all the other bodies of the system put together, and it is this great superiority of mass that enables the sun to keep its attendant planets and satellites in order, to control their motions and to cause them to revolve in an orderly array of ellipses.

All the various bodies, planets and comets, revolve about the sun in ellipses, and in each and every case the sun is at one focus of the ellipse. Yet in actual shape these paths differ widely;

the orbits of the great planets are nearly circular; while those of many comets are extremely long, narrow ovals, reaching out to untold distances from the sun. The paths of the major planets lie very nearly in a single plane, the greatest inclination to the ecliptic being a trifle over 7°. On the other hand the paths of the comets lie in all sorts of positions, apparently without regard to order or arrangement. But, whatsoever the shape of the ellipse, howsoever it may lie in space, the body, be it planet or comet, travels about the sun always sweeping over equal areas in equal intervals of time.

Five of the larger planets, Mercury, Venus, Mars, Jupiter, and Saturn, are among the most brilliant objects of the heavens; they have been watched and their motions studied from pre-historic times. Two great planets have been discovered since the invention of the telescope, and these two, Uranus and Neptune, are invisible to the unaided eye. Together with the earth, these bodies are the eight important members of the solar system. They are ranged about the sun in the order of their periodic times, or the lengths of their respective years. Mercury lies nearest the great central orb, then come, in order, Venus, the Earth, Mars, Jupiter, Saturn, Uranus, and finally Neptune, the outermost planet, some thirty times as far from the sun as the earth. Mercury requires but eighty-eight days to traverse its path about the sun, Neptune nearly one hundred and sixty-five years.

Now there is a direct relation between the distances of the planets from the sun and the lengths of their respective years. This rather complicated relation was discovered by Kepler, and is known as the third law of planetary motion. It may be stated as follows:

"The squares of the times of revolution of any two planets about the sun are proportional to the cubes of their mean distances from the sun."

This law is shown by the figures in the following table:

Planets	Distance	Cube of Dist.	Period	Sq. of Period
Mercury,	0.387	0.058	0.241	0.058
Venus,	0.723	0.378	0.615	0.378
Earth,	1.000	1.000	1.000	1.000
Mars,	1.524	3.540	1.881	3.538
Jupiter,	5.203	140.8	11.86	140.66
Saturn,	9.539	868.0	29.46	867.9

In this table the distances and times are those known to Kepler and are expressed in terms of the corresponding distance or periodic times of the earth. The numbers in the columns headed "cube of distance" and "square of period" are identical within the limits of practical error. Now this law of Kepler allows one to accurately compute the distance of a planet from the sun, as soon as its period is known. Celestial distances are almost impossible to accurately measure; they can only be determined indirectly through the slight shifting effects of parallax; while periods of time can be found with extreme precision. By

watching a planet through a long period of time, it will be found to return again and again to the same position relative to the sun, and this period, the length of its year, can be obtained with great accuracy.

It must be noted, however, that this law of Kepler does not give the distance in miles; but only in terms of some other distance, that of the earth from the sun for example. From this law the relative sizes of the orbits can be determined, and a correct map of the solar system may be drawn, but the scale of the map will be unknown. In order to find the scale of the map, to find the various distances in miles, one distance on the map must be measured. This is measured by means of parallax, and the distance between Mars and the earth, as has been seen, is the best for this purpose.

To convey an accurate idea of the dimensions and relative distances of the various bodies of the solar system by means of a chart or diagram is well-nigh impossible. The mechanical contrivances, or orreries, which purport to show the motions of the planets, are more than useless, for they cannot be constructed on anything like an approximate scale. A general impression as to the distances and motions of the planets may be gathered however from the following illustration. On the top of City Hall, New York City, place a great spherical lantern, or search-light, twenty feet in diameter to represent the sun;

then Mercury will be represented by a small plum, on the circumference of a circle 820 feet radius, or at the corner of Broadway and Thomas Street; Venus by an orange at the corner of Leonard Street, 1550 feet from City Hall; the earth by a large orange at White Street, 2150 feet distant; Mars by a good-sized plum at Grand Street, three-fifths of a mile away; Jupiter by an ordinary library globe two feet in diameter, placed two miles away in the middle of Madison Square; Saturn by a slightly smaller globe in the office of the new Plaza Hotel at 59th Street, some four miles from the starting-point; Uranus by a football on the Athletic Field of Columbia University at 116th Street, and Neptune by a large-size toy balloon in Bronx Park, a little over twelve miles from City Hall. In its orbit about the central luminary, this toy balloon, representing Neptune, will pass over the town of Hackensack, the cities of Passaic, Orange, and Newark, over the hills of Staten Island and the sands of Rockaway Beach, returning by way of Jamaica and Flushing, finally crossing the East River at Whitestone. To imitate the motions of the planets in these orbits Mercury must move at the rate of three feet an hour; Venus not quite two feet per hour; the earth nineteen inches; Mars fifteen inches, Jupiter eight, Saturn six, Uranus four, and Neptune only about three and a half inches per hour. This latter planet would require 165 years to complete its circuit of New York.

The Planets

In physical condition the various planets differ as widely as they do in size. Three of them, Mercury, Venus, and Mars, are not unlike the earth in size and in general characteristics. They are, in all probability, solid, cool bodies similar to the earth, and like the earth surrounded by atmospheres of cool vapours. The outer planets, Jupiter, Saturn, Uranus, and Neptune, on the other hand, are huge bodies, many times the size of the earth, and more nearly resembling the sun than the earth in their physical characteristics. They are globes of gases and vapours, so hot as to be nearly, if not actually at times, self-luminous. They may each contain a small solid nucleus, but the great bulk of these bodies consists of an immense gaseous atmosphere, filled with minute liquid particles; the whole at an extremely high temperature.

Of the actual surface conditions on Venus and Mercury little is definitely known. Mercury is a very difficult object to observe on account of its proximity to the sun. It is never visible at night; it must be examined either in the twilight just before sunrise or after sunset, or in full daylight. In either case the glare of the sun renders the planet indistinct, and the heat of the sun disturbs our atmosphere and makes "good seeing" extremely rare. The surface of the planet is probably rough and irregular, and not unlike that

of the moon, and to make the resemblance to the moon more complete, Mercury has little or no atmosphere. Still further, Mercury rotates upon its axis once in eighty-eight days; its day and its year are of the same length. Thus the planet always presents the same face toward the sun, and on that side reigns perpetual day, and on the other perpetual night, a night of unbroken, unimaginable cold.

Venus resembles the earth more nearly than any other heavenly body. It is almost a counterpart of the earth in density and in size; its diameter being 7830 miles, or only 120 miles less than that of our planet. Venus is shrouded in deep banks of atmospheric clouds, which effectually veil her mysteries from our gaze. Her atmosphere is considerably more dense than that of the earth, and the sunlight is reflected from the upper surface of clouds and vapours, and reaches our telescopes without ever having penetrated to the actual surface of the planet. Some few semi-permanent markings on the planet's disc have indeed been noted from time to time, but whether these were cloud forms or the tops of mountains, is not fully established. There is no clear, undisputed, evidence that any particular one of these markings really forms a permanent feature of the surface.

More is certainly known about Mars than about any other heavenly body, and yet very little is actually known in regard to the conditions on the surface of that ruddy planet. In many

respects, however, Mars is a miniature earth; its diameter is 4210 miles, and its surface area is a little more than the total area of dry land upon the earth. Like the earth, Mars rotates about an axis inclined to the plane of its orbit, and the length of a Martian day is very nearly equal to one of our own. The mean of a number of the best modern determinations gives the length of one complete axial rotation of Mars as equal to $24^h\ 37^m\ 22^s.65$ and this makes each Martian solar day $24^h\ 39^m\ 35^s$ long.

Mars is surrounded by a very light and transparent atmosphere, and through this many striking and permanent features of the planet's surface are visible. The most noticeable of these are the brilliant white "polar caps," first recognised by Sir William Herschel in 1784. These caps alternately wax and wane with the changing seasons on Mars; during the long winter in the northern hemisphere, the cap at the north pole steadily increases in size, only to diminish during the next summer, when it is exposed to the direct rays of the sun. The behaviour of the polar caps proves without question the presence in the Martian atmosphere of vapours, which are condensed and precipitated by cold and which are evaporated by heat. These vapours are in all probability water vapours, and the caps some form of snow and ice, or possibly hoar-frost.

Outside the polar caps the surface of the planet is rough, uneven, and shows a mass of faint

SOUTH

PLATE VI

Drawing of Mars by E. E. Barnard, Made with the 36-inch Lick Telescope

detail, which appears differently to different observers. The main part of the disc appears a deep, ruddy colour, and against this orange background darker, grey-green spots and markings are seen. The lighter portions were formerly thought to be continents and the darker portions seas and oceans. It is now known, however, that the grey spots and markings are not seas, that they are really rough and irregular portions of the surface, that in fact there is practically no free water on the Martian surface. Mars is a dry planet. In just what way, however, these dark markings differ from the lighter coloured portions of the surface is not definitely known. By some investigators the orange hues are called desert sands and the dark markings irrigated plains. But such an artificial explanation is not necessary. There are in all likelihood different coloured rocks and soils on Mars, just as there are on the earth and on the moon. The deep red clays of New Jersey are radically different from the dazzling sands of the coast. The moon, which has neither air nor water, has light and dark patches.

Some of the darker markings appear to be long, straight, streaks. They are the so-called "canals," or channels, so named by Schiaparelli, who first noted them in 1877. For many years the reality of this discovery was doubted, but during the last few years many astronomers have observed the more prominent channels and some few of them have even been photographed. To-day the reality

of some few of the more prominent of these markings cannot be doubted. These broad channels, however, must not be confused with the system of fine, sharp, lines, now so prominent in the drawings of Flammarion and Lowell. The existence of these latter, the "canals" of Lowell, is, at least, open to serious question. Keeler, Campbell, and Barnard, using the largest and best telescopes in the world, have failed to observe them. According to Newcomb the canaliform appearance as depicted by Lowell "is not to be regarded as a pure illusion on the one hand, or an exact representation of objects on the other. It grows out of the spontaneous action of the eye in shaping slight and irregular combinations of light and shade, too minute to be separately made out, into regular forms."

The term canal, which has been applied to these indistinct markings, is an unfortunate one. The word implies the existence of water and the presence of beings of sufficient intelligence and mechanical ability to construct elaborate works. Flammarion in France and Lowell in this country assert that the word is correctly used, that these markings are really canals and that Mars is inhabited. But this is pure speculation and imagination. There is no conclusive proof that the lines, or canals, really exist as drawn; they are probably to a large extent illusions and figments of defective vision. There is not the slightest trace of any artificial work on Mars itself;

the artificiality is in the drawings, not in the planet. There is no evidence of any kind, pointing toward the existence of conscious life upon Mars. So far as we know it is not impossible for life of some form to exist there, but, if it does, we, as yet, do not know it.

The Stars

Sharply distinguished from the planets, or wanderers, are the "fixed" stars. These appear as mere points of light and always maintain the same relative positions in the heavens. Thousands of years ago, when the shepherds first trod the plains of Chaldea, the "Great Dipper" hung in the northern sky just as it hangs to-night, and as it will hang for thousands of years to come. Yet these bodies are not absolutely fixed in space. In reality they are all in motion and in rapid motion, some moving one way, some another. It is their immense distance from us that makes this motion inappreciable. Great as may appear the distance between the sun and the earth, yet this distance shrinks to a mere nothing as compared to the distance from the earth to a star, and from star to star. The nearest star is more than 200,000 times as far from us as is the sun. Expressed in miles, the figure representing this distance would be so enormous as to convey no distinct impression. A special unit has, therefore, been invented—a unit represented by the distance

traversed by light in one year. In one second light travels over 186,000 miles, nearly eight times around the earth's equator; in 8⅓ minutes light reaches us from the sun, covers the distance that it would take the *Mauretania* over four centuries to travel. Yet the nearest star is over four "light years" distant; it is so far away that it requires over four years for its light to reach us. When we look at the stars to-night, we see them, not as they are, but as they were years, even centuries ago. Polaris, the North Star, is distant some sixty light years; had it been blotted out of existence at the time the *Monitor* fought the *Merrimac*, we should as yet be unaware of the catastrophe.

The stars appear to be countless in number, yet, in reality, comparatively few can be distinguished by an unaided eye. In the whole heavens there are not more than six thousand which are bright enough to be clearly located and counted, and less than one half of these can be seen at any one time. Even on a clear, moonless night, it is doubtful whether the sharpest eye can detect more than two thousand separate and distinct stars. One may, indeed, be conscious of a background of light, or of luminous points, but no eye can separate it into clearly visible stars. Where the eye fails, however, a small telescope even is successful. With an opera-glass the whole sky seems filled with stars, and over one hundred thousand may be seen and counted; with the Yerkes, or other great,

telescope a hundred millions may be distinguished. The photographic plate reveals countless myriads that otherwise would for ever remain invisible and unknown.

Now each and every one of these stars is a *sun;* is a vast globe of gas and vapour, intensely hot and in a continuous state of violent agitation, radiating forth heat and light; its every pulsation felt throughout the universe. So closely, indeed, do many of the stars resemble the sun, that the light which they emit cannot be distinguished from sunlight. Some of them are vastly larger and hotter than our sun, others smaller and cooler, yet may our sun be regarded as a typical star, and from our knowledge of it, we can form a tolerably correct conception of the nature and constitution of the other heavenly bodies.

The stars differ among themselves in brightness, and they may be classified accordingly. The early Greek astronomers divided the 1100 or so stars which they knew into six groups, or "magnitudes." In the first group they placed a dozen or more of the brightest stars; in the sixth group was the mass of stars just visible to the unaided eye, while the stars of intermediate brightness were arbitrarily classed as of the second, third, or fourth magnitude. The word magnitude as used in this connection refers to brightness only, and the greater the magnitude the fainter the star. Small magnitudes mean bright stars, large magnitudes faint stars. In these early groupings

of the stars, the comparison of the brightness of one star with that of another was a mere eye estimate—one star appeared brighter than the other. To-day, however, there are instruments by means of which this comparison can be quickly and accurately made, and the relative amount of light received from the two stars measured with precision. A definite scale of magnitude has, therefore, been adopted and is now in use in all publications.

On this absolute scale of star magnitudes, a star of the first magnitude gives out $2\frac{1}{2}$ times ($\sqrt[5]{100}$) as much light as a star of the second, and this star in turn is $2\frac{1}{2}$ times as bright as one of the third magnitude. The ratio between the light of a star and that of another, one magnitude fainter, is thus constant and equal to $2\frac{1}{2}$. Hence a sixth magnitude star is exactly 100 times fainter than a star of the first magnitude; one hundred stars of the sixth magnitude are required to give us as much light as we receive from a single star of the first order. Each increase of five magnitudes corresponds to a diminution of a star's light by 100. It would require 10,000 stars of the eleventh and 1,000,000 stars of the sixteenth magnitude to equal the brightness of a star of the first.

With the adoption of this scale and the use of accurate photometers it became necessary to subdivide the magnitudes, to use fractional and negative magnitudes. The brightness of different stars shades gradually from one magnitude to

another; one star may be a trifle fainter than the second magnitude, yet not so faint as the third. Such a star would be called of the $2\frac{1}{3}$ magnitude and to cover these cases it is now usual to give the brightness of a star to the nearest tenth of a magnitude. Still further, some of the old first magnitude stars have been found to be considerably brighter than the standard, and hence on the scale their magnitude, or brightness, must be expressed by a number smaller than unity. Arcturus, for example, gives out nearly $2\frac{1}{2}$ times more light than Aldebaran, a typical first magnitude star. Arcturus must, therefore, be of the 0th magnitude. Again Sirius is still brighter, giving out between nine and ten times as much light as Aldebaran; it is therefore 2.4 magnitudes brighter, or is of the -1.4 magnitude.

Now these wide variations in the brightness of stars are due partly to actual differences among the stars themselves, and partly to varying distances. If all the stars were alike then those which were farther away would be fainter, magnitude and distance would have a definite relation, and the relative distances could be accurately determined from the magnitudes. This is not the case, for "one star differeth from another star in glory." Some of the more brilliant stars are far more distant than some of the fainter ones. There are stars near and remote, large and small, and an apparently faint star may in reality be far larger and hotter and more brilliant than a star of the

first magnitude. The actual distances of a number of stars have been measured, and while the brilliant α Centauri is found to be our nearest neighbour, a small, faint star, invisible to the unaided eye, is next in order of distance. Sirius, the most dazzling star in the heavens, is fifth. With their distances known the actual amount of light emitted by the various stars can be calculated. It is found that Sirius is vastly more luminous than the sun, giving out over forty times the light of our central orb. Our nearest neighbour, α Centauri, is about twice as bright as the sun, Vega more than fifty times, and the faint Pole Star is in reality a vast globe giving out nearly one hundred times the light and heat of the sun. If the sun were removed to the distance of the nearest star, it would appear about as bright as Polaris, and not one of its retinue of planets could be detected; if it were removed as far as the Pole Star, it would no longer be visible to the unaided eye. The sun, as a star, is relatively small and feeble.

In very early times the visible stars were divided into fanciful groups or "constellations." Many of these groups originated in pre-historic times and have some connection with the mythology of the ancient peoples. Queer and impossible figures of men, women, and animals were represented in these fictitious groups. Yet the stars themselves bore not the slightest resemblance to the figures assigned; there is no imaginable

reason why the constellations should be named or bounded as they were. The constellation known as the "Great Bear" resembles a dipper with a bent handle, and is to-day familiarly known as the "Great Dipper." These artificial groups, and the fanciful names and pictures assigned to them, are of no practical use to-day. The astronomer never uses them; they only cumber up the star charts and render the study of the actual heavens more difficult.

Further these star groups are only apparent, not real. Two stars which appear close together in the heavens, may in reality be immeasurably far apart, the one being comparatively near the earth, the other far out on the confines of space. The seven stars which form the Great Dipper are really separated by immense distances, and some are travelling in one direction, some in another. The present dipper-like appearance is temporary; many hundreds of centuries ago the dipper was not, and centuries hence it will not be. Of course, in some cases, stars which appear close together, really form a sort of family: some sixty or seventy stars of the Pleiades are all travelling together along similar paths, and are all members of a definite group bound together by the tie of mutual gravitation.

To-day a star is located in the heavens by means of its right ascension and declination; the astronomer knows and locates a star just as a mariner knows and locates a rocky islet in the

broad waters of the ocean, by its geographical position. Comparatively few of the stars have individual names, or any designation other than the figures which give them locations on the celestial sphere. Often they are referred to, however, by the current numbers which they happen to have in some well known catalogue. Thus for example a star may be spoken of as 20,864 Lalande, or another as 1084 Yarnell. The catalogues in question give the right ascensions and declinations of these stars, and thus they may be completely identified.

Most of the brighter stars, however, have special names, which have come down from prehistoric times. Aldebaran, Vega, are Arabic in origin, while Sirius, Procyon, and others are of Greek or Latin derivation. Nearly all the visible stars are designated by Greek letters, or numbers, prefixed to the name of the constellation in which they are found. These letters and numbers were intended to be assigned in the order of brightness of the various stars. Thus α Tauri is the brightest star in the constellation, β Tauri, the second star, and γ, the third brightest. In some cases errors were made in estimating the relative brightness of the stars, and the letters were assigned to the wrong star. Castor is less bright than Pollux, yet the former is known as α and the latter as β Geminorum. All the bright stars thus have more than one name, they have their own specific name, as well as the constellation letter. Aldebaran is also α Tauri.

Navigational Stars

Only the brighter stars can be used in navigation. So much light is lost in the double reflection in the mirrors of the sextant that stars fainter than the third magnitude can seldom be observed. This reduces the number of stars available for navigation to within very narrow limits, for there are only 142 stars all told which are of the third magnitude or brighter. The Nautical Almanac gives a list of 150 stars, which may be used, but, as a matter of fact, the list might be reduced to some fifty or sixty without serious detriment to the practical navigator. About thirty of these are of the second magnitude, or brighter, and are always readily found.

It is not difficult to learn to know the thirty or forty brighter stars, so that they can be recognised at any time. To aid in locating the stars many different star charts and atlases have been published, but most of them are so elaborate that they confuse rather than help the novice. The simpler the chart, the fewer stars it pretends to locate, the better for practical purposes. Again all charts are of necessity printed on a flat surface, and such a surface can never represent in their true relations all parts of a sphere. A chart which covers a large portion of the heavens will give a distorted idea of distances or directions in some part of the sky and must be used with caution. There are a few groups of stars which form striking

geometrical figures of one kind or another, and these are easily located and serve as starting points in the search for individual stars. Of these groups the Great Dipper is the most prominent in the northern sky, and beginning with this the other constellations can one by one be located.

When the groups, or constellations, are not known, then any individual star can readily be found by means of its tabular right ascension and declination. These co-ordinates of the heavens correspond very closely to ordinary longitude and latitude on the earth's surface; declination is identical with latitude, while right ascension is always measured to the east, entirely around the sphere, instead of both to the east and west as on the earth. Right ascensions are measured from the vernal equinox and unfortunately this point is not marked by any prominent celestial object. Further, owing to the annual motion of the sun through the heavens, different parts of the sky are brought into view at different seasons of the year. The equinox and each and every star are thus brought to the meridian some four minutes earlier each day. In March, when the sun is on the equator, the equinox is overhead at noon and all that portion of the sky is invisible. By the middle of the summer, the sun has so moved that the equinox is brought to the meridian in the early morning hours, by September it has gained twelve hours on the sun and culminates at midnight. A given right ascension is not,

therefore, directly associated with any fixed hour of the solar day. But, as will be more fully explained in another chapter, sidereal time is counted from the transit of the vernal equinox, and the sidereal time at any instant is equal to the right ascension of the point then on the meridian. At three o'clock sidereal time, every star whose right ascension is three hours, or 45°, will be found on the observer's meridian, and each succeeding sidereal hour will bring to the meridian stars whose right ascensions are four, five, and six hours respectively. Thus by merely noting the time on a sidereal chronometer, the position of any star in the heavens can be at once determined. If the star's right ascension is less than the sidereal time, the star will be found in the west, if the right ascension be greater, the star will be to the eastward of the meridian: and the star will be found just 15° to the west or to the east for each hour of difference between the right ascension and the sidereal time.

When the general east and west direction of any star has thus been determined, its north and south position can at once be found from its declination. This is given in degrees and it measures the angular distance of the star north or south from the equator, and the general position of the equator in the sky is always readily found.

Thus, in locating a star, a knowledge of the sidereal time is essential. For any one making a practice of star observations a good sidereal

chronometer is therefore a great convenience and it is surprising that they are not more often carried on well-equipped vessels. Even when a regular sidereal chronometer is not available, a watch or a hack chronometer might be regulated to run fairly well on sidereal time, and such an instrument would be of great use in finding and identifying stars and in stellar observations generally. But, when no such watch or chronometer is at hand, the sidereal time can be found by simple calculation from the mean time; the data for the calculations being found in the *Nautical Almanac.*

CHAPTER V

THE MAKING OF AN ALMANAC

TO many the word almanac brings up the picture of a small pamphlet, profusely illustrated with queer signs and symbols, and full of interesting information in regard to history, politics, and the state of the weather. The true almanac, however, differs radically from this picture; it is a tool of the navigator and of the astronomer. It is a book full of uninteresting and meaningless tables for the ordinary reader, but a book without which the *Lusitania* could never have made her record-breaking passages across the ocean. The nautical almanac of to-day is an accurate chart of the ever-varying heavens; its tables furnish the navigator with detailed information as to the positions of the sun, the moon, and the planets on each and every day and hour of the year, it furnishes the astronomer with a mass of detail which simplifies the every-day routine of his observations. It is a book of exact information, prepared by astronomers for the use of astronomers and for the guidance of travellers by land and by sea.

From the data contained in this almanac are

prepared and made all the minor almanacs and calendars; the maker of nautical instruments reprints such portions as are necessary for an ordinary coasting voyage, and makes such an emasculated almanac serve as an advertisement of his business: the purveyor of patent medicines dilutes a small portion of this valuable book with antiquated signs and mystifying symbols and mixes with the concoction gratuitous medical advice and sage remarks upon the weather.

How the information contained in the original, *the nautical*, almanac has been gathered together, how it has been tabulated and made available for immediate use, it is the purpose of this chapter to explain.

There is no record as to when the first almanac was made: tables, somewhat similar to the ordinary farmer's almanac of to-day, were used in Greece several centuries before the Christian era. These tables gave for a series of years the dates of the changes of the moon, the times of the rising and the setting of well-known constellations, and, to assist the farmers still further, they contained weather predictions, and assigned the best dates for planting the crops and reaping the harvests. But even these early examples of almanac making were preceded by other tables: almanacs of a still cruder form. The Babylonians and the Egyptians, some five thousand years ago, some thirty centuries before our era began, knew the approximate length of the year and possessed tables, or calen-

dars, by which the dates for feasts and festivals were regulated.

The first essential in almanac making is an accurate measure of time. This is furnished by the daily rotation of the earth on its axis, by the regular recurrence of day and night. This rotation is perfectly uniform and the day is of the same length now as it was when the shepherds first watched the stars from the plains of Chaldea. From a comparison of the times at which eclipses, transits, and other astronomical phenomena occurred in bygone years, it can be shown that the day has not changed in the last 2000 years by so much as the one hundredth part of a second. The day is the unit of time, it is the basis of all calendars, of all astronomical calculations.

But, while the day is thus a natural and a perfectly accurate measure of time, it is too small a unit when we come to measure long intervals. We do not ordinarily reckon the fortune of a Rockefeller by pennies. Now there are two other and longer natural periods by which time might be measured: the month and the year. Unfortunately, however, neither the month, nor the year, contains a whole number of days; the month consists of some $29^d\ 12^h\ 44^m\ 2^s.684$; the year of some $365^d\ 5^h\ 48^m\ 45^s.51$. It is, therefore, a difficult matter to use these different units in a common system, or in an ordinary calendar, by which the passage of time and the recurrence of the seasons are recorded.

By the early Greeks many attempts were made to arrange a satisfactory calendar, to make a year consist of a definite number of days and months. These attempts were only partially successful, for they wished each new month to begin on the day when the new moon first became visible. As twelve lunar months consist of a trifle more than 354 days, while a year is over $365\frac{1}{4}$ days long, it is evident that the new moon will not occur at the same date on successive years. In 1907 the first new moon of the year was on January 13th, in 1908 on January 3d. The Greeks tried to rectify these troubles by arbitrarily putting in extra days, when necessary. So mixed were their calendars, that in 423 B.C. Aristophanes in his play *The Clouds* makes the Moon complain

Yet you will not mark your days
As she bids you, but confuse them, jumbling them all
 sorts of ways.
And, she says, the Gods in chorus shower reproaches
 on her head,
When in bitter disappointment, they go supperless
 to bed,
Not obtaining festal banquets, duly on the festal day.

Julius Cæsar, in the year 46 B.C., first put the calendar on a proper basis. He neglected the moon entirely and made the year consist of $365\frac{1}{4}$ days. This was done by making the ordinary year contain 365 days and in each fourth year an extra day was added to the month of February.

This was a great advance, and the most perfect calendar yet invented, but the year is a little ($11\frac{1}{4}$ minutes) less than $365\frac{1}{4}$ days long, and so at the end of 128 years the difference between the calendar and the true year would amount to one day. Now this would have done no harm; at the end of some 230 centuries, some 20,000 years from now, the seasons would have completely shifted around, winter occurring in the month called July, and the hottest part of the year being in January and February. But the priests thought this gradual change in the seasons most deplorable, the Church festivals would be ruined if they did not happen in appropriate weather; so Pope Gregory, in 1582, complicated the simple Julian Calendar, by arbitrarily adjusting it by some ten days, and by decreeing that in the future three leap-year days should be omitted every four centuries: thus the years 1700, 1800, 1900 were not leap years, but 1600 was and 2000 will be. Thus, in order to suit the fancy of the Church, the beautifully simple calendar of Cæsar was disarranged and all dates and calculations complicated.

While the Church calendar was thus being developed, the true almanac was also being slowly evolved. To a traveller on an unknown or unmapped portion of the earth, astronomy offers the only means of finding his real position: to the navigator the heavenly bodies serve as guides across the pathless oceans. To meet the demands of commerce, astronomers made crude

tables and charts of the heavenly bodies, and furnished the navigator with the means of pushing his voyages to distant lands. After the discovery of America, voyages became longer and more frequent and the demand for accurate astronomical tables became urgent; the old tables, the old methods, which had been in use for centuries and which sufficed for short coasting voyages, no longer served the purpose. New tables were essential to the progress of the world. England was the mistress of the seas, her ships were in every port, but the methods of the navigator were crude and the government offered a large reward or prize for a satisfactory method of finding a ship's place at sea. In the year 1675, a Frenchman, Le Sieur de St. Pierre, claimed the reward with a method of finding "the longitude at sea from easy celestial observations." The greatest difficulty in finding the longitude at this time was the lack of accurate time, chronometers being unknown. Now the moon may be regarded as the hand of a great celestial clock, travelling around the dial once each month, the stars and planets marking the hours and the minutes. St. Pierre showed that by measuring the position of the moon among the stars, the time could be easily found, and from the time the longitude was at once known. Unfortunately this method depends upon an accurate knowledge of the path of the moon and of the speed with which it travels that path, or, as astronomers put it, upon

PLATE VII

Greenwich Observatory in Flamsteed's Time

the possession of accurate "tables of motion" of the moon. With Flamsteed, as adviser, a Committee investigated St. Pierre's claims, and found the method good, but the "lunar tables" upon which rested its practical application were faulty, often containing errors as large as 12 minutes. The longitude of a ship might, therefore, be out many miles.

The Commissioners rejected St. Pierre's claim to the reward, but, at the suggestion of Sir Jonas Moore, memorialised the King, advising the erection of an observatory for the purpose of making observations which might serve for the discovery of the longitude. Thus was started the Royal Observatory at Greenwich, Flamsteed becoming, on March 4, 1675, the first "Astronomer Royal," whose duty it was, according to the warrant of Charles II., "forthwith to apply himself with the most exact care and diligence to rectifying the tables of the motions of the heavens, and the places of the fixed stars, so as to find out the much desired longitude of places for the perfecting the art of navigation." The government, however, was not over liberal, Flamsteed's net salary being only £90 a year, and, although an Observatory building was erected, no instruments were furnished. Flamsteed was practically obliged to build instruments at his own expense, and for some thirty years he maintained the Observatory out of his private income, for he fortunately inherited a little money.

Although Greenwich was founded "for the perfecting the art of navigation," yet nearly one hundred years elapsed after the appointment of Flamsteed before the work of the Observatory became of direct value to the navigator. To Maskelyne, the fifth Astronomer Royal, is due the foundation of "The Nautical Almanac." In 1761, at the suggestion of the Royal Society he made a voyage to St. Helena to observe a transit of Venus. On this voyage he made many observations and thoroughly tested the method of "lunars," and in a later voyage to Barbadoes he tested the "chronometer method," using the recently invented time-keeper of John Harrison. These voyages convinced him of the necessity of furnishing the navigator with tables and aids for the rapid reduction of observations. In 1763, therefore, he published the *British Mariner's Guide*, a handbook of tables for the determination of the longitude at sea, which revolutionised the practice of navigation. Upon his appointment as Astronomer Royal he enlarged and improved this little book and made it an official government publication under the name of the *Nautical Almanac*. The value of this book and the accompanying tables was so instantly recognised that 10,000 copies were sold within a short time.

The *Nautical Almanac* thus begun by Maskelyne has been continued by his successors in the office of Astronomer Royal. "Tables of the motions of the heavens" have been perfected, and to-day

Plate VIII

Nevil Maskelyne, the Founder of the Nautical Almanac

the results of centuries of scientific investigations are instantly available to the navigator. The governments of France, Germany, and America have established great observatories to aid the navigators of their countries and to furnish them with the indispensable positions of the heavenly bodies. From the *American Ephemeris and Nautical Almanac* the position of the sun for any time can be instantly found; and found with an accuracy far exceeding that with which a navigator can make an observation. These books are printed many years in advance, and give not only the position of the sun for every day, but predict also the positions of the moon, the planets, and the stars. Without this work of the astronomer, the vast fleets, that daily traverse the oceans in safety, would be obliged to lie at anchor, or to skirt the shores and crawl slowly from port to port.

The making of an almanac begins with the direct study of the heavens, begins in an astronomical observatory. In fact the entire "making" may be divided into three distinct parts.

1. The collection of data by the direct and minute study of the heavens. This work is carried on in the great governmental observatories and involves the systematic observation and measurement of the varying positions of the sun, the moon, and the planets.

2. The study of these data, the deduction therefrom of the laws which govern the motions of

the planets, and the formation of tables which express these motions. This is the work of the mathematical astronomer and it is carried on at a desk with pen, ink, and paper.

3. The calculation, from these tables, of the positions for given dates of the sun and the planets. This is the routine work of computers and is carried on in the governmental bureaus.

For the first division, the collection of data, great and costly observatories are necessary. For this purpose, was founded the Royal Observatory at Greenwich, and in our own country the Naval Observatory at Washington. In these institutions the practical, the observing astronomer, day after day and night after night, measures and plots the positions of the sun, the moon, and the planets among the stars. He studies and is concerned with their actual motions; delicate instruments are his to handle and to observe with.

The fundamental instruments of astronomy of position are the meridian circle and the clock. With these are determined the positions of the bodies in the heavens. Just as on the earth we locate a mountain, a city, or a town by its latitude and its longitude, by its distance north or south of the equator and its distance east or west of Greenwich, so in the heavens the positions of the sun, of Jupiter, and of the stars are known when their distances north or south of the celestial equator and their distance east or west from the vernal equinox are known. In the heavens, de-

clination corresponds to latitude, right ascension to longitude on the earth, and the vernal equinox to Greenwich. The earth rotates upon its axis with perfect uniformity, and minute after minute, hour after hour, different portions of the heavens are brought on our meridian. If we have a perfectly running clock and note the time when the vernal equinox crosses overhead to-day, then exactly 24 hours (sidereal) later it will again transit. If Jupiter comes to the meridian one hour after the equinox, then we know that Jupiter is one hour, or $\frac{1}{24}$ of the circumference of the heavens, east of the equinox. Its longitude, so to speak, will be 15° east. Thus by noting the times at which various bodies cross the meridian, their east and west positions in the heavens can be accurately determined, their right ascensions found.

For this purpose an accurately moving clock is essential. The standard clock of an observatory, therefore, is one of its chief instruments, and as the rate of such a clock is affected by tremors and jars, by changes of temperature, and by the varying weight of the atmosphere, it is usual to mount astronomical clocks in specially built rooms—rooms that are free from outside disturbances and that can be kept, winter and summer, at an even temperature. In such a constant temperature vault a good clock, properly mounted and well cared for, should run for three or four years without being stopped, and should never

vary its rate by so much as one-fifth of a second per day during the entire interval.

With the clock must be used the "meridian circle" to indicate the instant of a body's crossing the meridian. In principle this instrument is simplicity itself. It is a telescope so mounted that it revolves in the plane of the meridian, the line of sight passing in a great circle through the pole, the zenith, and the south point. The instrumental meridian is marked by a spider line in the field of view, and the time of passage of a star across this is noted on the clock. If the instrument be perfect and in perfect adjustment, then this instant will mark the meridian passage of the star. But no matter how firmly mounted and how carefully made, an instrument can never be in precise adjustment. Jars and changes of temperature affect it, one pier may be expanded a little more than the other, the instrument may have a tilt, and the line of sight describe an inclined circle on the sky. But with care an instrument can be kept almost in the meridian, and by observing three or four stars as they cross the meridian, some in the north and some in the south, it is possible to determine the exact departure of the instrument from perfect adjustment, and to correct any other observation for the errors introduced by this maladjustment. That is, from observations more or less inaccurate, but honest, made with an instrument more or less maladjusted, but firm, it is possible to obtain a close

approximation to a perfect observation made with a perfect instrument in perfect adjustment. The time of transit of a star across the meridian can be found to within $\frac{1}{20}$ of a second.

To find the declination of a body, or its distance north or south of the equator, the meridian circle is provided with a finely graduated circle, which lies in the plane of the meridian and revolves with the telescope. With this the angular distance of any body north or south of the zenith is found, and as the distance of the zenith from the equator is known (it is the latitude of the observatory), simple addition or subtraction gives the declination of the observed body.

Besides these two fundamental instruments, an observatory must be provided with an immense amount of accessory apparatus. As from the necessity of being firmly mounted a meridian circle is a small instrument, faint objects cannot be observed with it. The great lenses, the Washington, the Lick, and the Yerkes, with which the positions of the fainter bodies are measured, are all mounted equatorially, mounted so that they can be turned toward any part of the heavens. But the measures made with such a telescope are not fundamental; they must be referred to and based upon a prior measure made by a meridian circle. With the meridian circle are determined the positions of thousands of stars scattered in all portions of the heavens, and these serve as milestones to mark the paths of the planets. With

the equatorial, the position of a planet on any day is found in reference to one of these known stars, its distance from the star measured, and, using the star as a known point in the heavens, the position of the planet may be easily computed.

The work of an observing astronomer is not finished as soon as he has noted a time on his clock, or measured the position of a planet from a star. Far from it, for every hour he spends at the end of a telescope, he, or an assistant, must spend four hours at a desk with pen, ink, and paper, figuring out his results and finding out exactly what he did measure. To the raw or crude measure as it is put down in the observing note-book, many corrections must be applied. We have seen that no instrument is ever in perfect adjustment—these instrumental errors must be found and their effects upon the measured position calculated and allowed for. But over and above all this, with even a perfect instrument in perfect adjustment, we could never see a star where it really is. When we think we see the sun just rising above the eastern horizon, it is really below that horizon and out of direct sight. The rays of light are bent as they pass through our atmosphere; instead of seeing in a straight line, we look along a curved path. Now the amount of this refraction, or bending of the ray of light, varies; it is very small for bodies nearly overhead, large for bodies near the horizon. It changes with the temperature of the air and varies with the rise and fall of

the barometer. All these varying atmospheric conditions must be noted by the astronomer and his crude measure corrected accordingly. Again it takes the light of the sun some eight minutes to reach us, and we see it now, not where it *is*, but where it *was* eight minutes ago. Light is four hours in coming from Neptune, the outermost planet.

When the observing astronomer has made all necessary corrections and finished these laborious calculations, he has a complete observation, the exact position of the sun or a planet on a given day and year. These observations are published in the astronomical journals and in the observatory's reports or year book. Great annual volumes are published by Greenwich and by the Naval Observatory at Washington, in which are recorded thousands of observed positions of the various heavenly bodies. So much for the collection of data, the work of an observing astronomer, and the first step in the making of an almanac.

The second step toward the completed almanac consists in collecting the data gathered in the different observatories all over the world, fitting the various observations together, deducing the laws of planetary motion, and forming tables which express those motions. This is the work of the theoretical astronomer; it requires the highest order of mathematical mind. The greatest scientists the world has ever seen have laboured on

these problems, Newton, Laplace, Le Verrier, Newcomb—have all shared in the present perfectness of these tables. The grandest scientific discovery of all ages, "the law of universal gravitation," was evolved by Newton from his study of the motions of the moon.

The difficulties of the subject will be better understood by a moment's consideration of the various paths travelled by the planets. Nearly three centuries ago Kepler, with the data then available, showed that these paths might be considered as ellipses and that the sun is at one focus. As a result of the law of gravitation, Newton proved that Kepler's statements must be modified, that while the average path of any one body may be an ellipse, yet the real, the actual path is not an ellipse. If there were but two bodies in the universe, the sun and the earth, for example, then Newton proved conclusively that the earth would for ever describe about the sun the mathematical curve known as an ellipse. If these two bodies were the only bodies in the universe, then if their relative positions were known at any time, their future history would be pre-determined and mathematicians could quickly calculate their positions for any future, or for any past, date.

While thus the problem of two bodies is completely solved, the case is different as soon as we have a system of three bodies or more to deal with. If there be in the universe three bodies—the sun, the earth, and a comet or another planet, then the

mathematicians can no longer solve the problem as to what paths the various bodies will describe. All they can do is to trace out the motions step by step. On a given day, the positions of the three

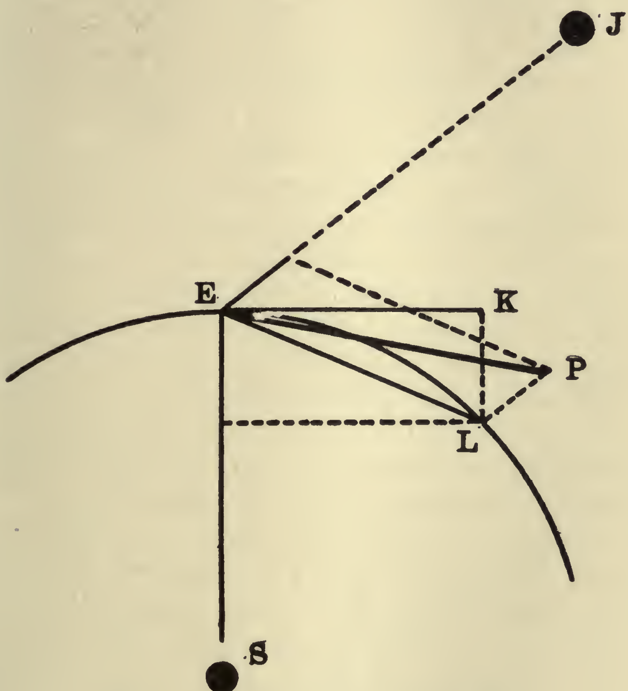

Fig. 5. The Problem of Three Bodies

bodies are as in this diagram. If the earth were the only body in the universe, it would move as indicated by the straight line EK. But during the time the earth is so moving, the sun attracts

it, pulls it out of its path, and causes it to describe a curve; and at the end of the period it is found to be at L. But Jupiter pulls the earth and hauls it out of its path, so that at the end of the period it is not at L, not in the position that it would be if the sun and it were the only bodies in the universe. It has been attracted and pulled by Jupiter to another position and appears at P. Knowing the positions of all the bodies to-day, the mathematicians can calculate how much the sun would pull the earth out in one direction, how much Jupiter would pull it out in another direction, how much Saturn would pull it out in a third direction, and thus they can predict where the earth will be to-morrow. Then knowing where all the bodies are to-morrow, they can go through the same process and show where they will be the next day, and so step by step, and very laborious steps they are, they can show where any one of the bodies of the system will be at a future date.

Fortunately in the case of the earth and the larger planets, the bodies which are of importance in navigation, their paths are so placed that they never approach closely one to another. They never depart far, therefore, from simple elliptic paths around the sun. The perturbations or disturbances in their motions are comparatively small. Instead, therefore, of being obliged to proceed step by step, the astronomer may assume that the average path of each planet is an ellipse, may find its approximate position for any time

on this assumption, and then compute for that date the effect of the various pulls. As the planets travel around and around in their orbits, they keep coming into the same relative positions: once every 399 days Jupiter, the earth, and the sun are in a straight line. At such times Jupiter

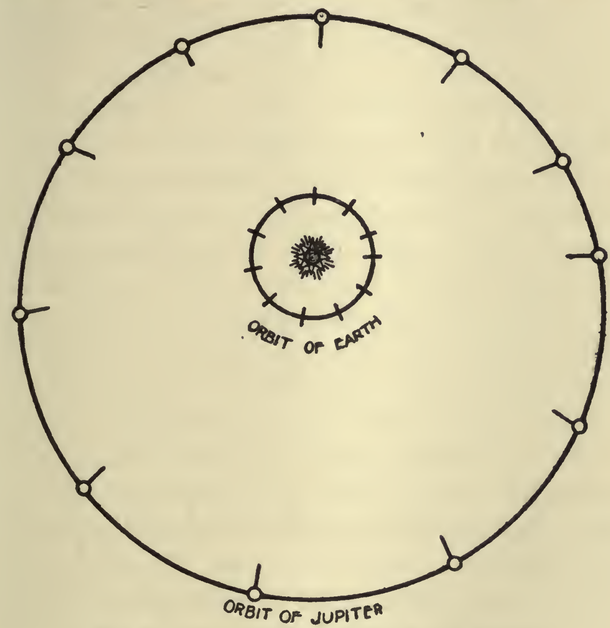

Fig. 6. The Orbits of the Earth and Jupiter

will pull against the sun, and will drag the earth from its simple path, the amount depending upon the relative distances between the earth, Jupiter, and the sun. If the orbits of both Jupiter and the earth were circles, this effect would be exactly

repeated every 399 days; the perturbation would be periodic, would repeat itself in regularly recurring periods of 399 days. But the orbits are not circular, they are elliptic; and so when Jupiter returns to opposition, the relative distances between the planets and the sun will have changed, and the perturbation of Jupiter upon the earth will not be the same as before; it will differ in amount. Thus the perturbation depends upon the part of the orbit in which the opposition happens; it will change in character and amount as the opposition falls in different seasons of the year. It is still periodic, it goes through regularly recurring changes, but the period is not the simple one of 399 days. The distance of the earth from the sun increases and diminishes again during one year, the distance of Jupiter from the sun fluctuates with its periodic time of eleven years (11.86), and thus the actual value of the perturbation depends upon the 399-day period, the 365-day and 11.86-year periods, and upon other modifying periods too complicated to mention here.

The entire disturbance in the motion of the earth, caused by Jupiter, can thus be separated into a series of periodic perturbations, and each one of these can be calculated and tabulated once for all. The formation of such tables taxes the skill of the astronomer to the utmost; intricate mathematical formulas must be developed, and long numerical computations carried out. It must

also be remembered that the orbits of the bodies are not known, they must be deduced from actual observations, and these orbits must be calculated before the tables can be computed. In finding the orbit of the earth, from which the *Tables of the Sun* were afterwards made, Newcomb used some 50,000 separate and distinct observations, each one of which had probably required hours of patient effort on the part of some observing astronomer. It took Hill and a computer ten years to form the tables of motion of Jupiter.

Newcomb's *Tables of the Sun* consist of 38 tables, and some 30 pages of explanations, the whole forming a quarto volume of 169 pages. And from these tables the position of the sun can be found for any date during some thirty-five centuries, or from 1200 B.C. to 2300 A.D. Similar tables are made for the various planets, those of Jupiter and Saturn being considerably larger and more complicated than those of the sun.

These tables are no sooner completed than they become the subject of investigation. Observations are made and the actual positions of the bodies compared with the theoretical positions as taken out of the tables. Each year the great observatories make long series of measures of the sun and the planets, and by these the tables are tested and corrected if necessary. While minute variations may be found by the accurate instruments of an observatory, yet the tables as prepared under the direction of Simon Newcomb are so

nearly perfect that for the practical purposes of the navigator they may be used for the next two or three centuries. That is, two hundred years from now an observer using a sextant will be unable to detect any difference between the actual place of the sun in the heavens and its place as predicted by Newcomb.

The third step in the making of an almanac is purely mechanical. A corps of clerks or computers is needed, supervised by one man of ordinary mathematical ability and intelligence. This bureau is provided with copies of the planetary tables and with plenty of blank paper. Then from the tables the positions of the various bodies are computed. Take the formation of the *Solar Ephemeris*, for example. The navigator needs to know the position of the sun at any time he may make a latitude or a longitude sight. The almanac gives the position for noon of every day, and these noon positions are found from the "tables." The finding of one such position of the sun requires the looking up some one or more figures in every one of the 38 tables and the writing down of some 235 figures. Each one of these figures is either taken from a table or is the result of an addition or multiplication; each one, therefore, requires a distinct mental effort.

While this work is mechanical and needs no astronomical training, yet it must be accurate. There are 235 chances of error in each position of the sun. To guard against this, the work is

done in duplicate: two computers carry on the work independently. Their results are compared, and if they do not agree, the work is gone over again and again until the error is found. But even with this precaution elaborate systems of checking are required, for errors will creep in, and the work when published must be free from every possibility of numerical error.

Fortunately it is not necessary to compute from the tables the position of the sun for every day of the year: the labour would be tremendous. The sun moves along its path smoothly and evenly, and if its position is found for every four days, the intermediate positions can be found by interpolation. That is, if its position is calculated for the 13th, the 17th, and the 21st of December, the rate at which it is moving between the 17th and the 21st can be found, and from this rate it is easy to find its position for the 18th, 19th, and 20th. Thus the actual labour of using the tables is much reduced, and this same method furnishes an additional check on the accuracy of each computation. For if the position as found for any day does not fit into a smooth curve, if it shows that the sun has suddenly moved faster or slower than it should, then it is safe to assume an error in the computation.

In the case of the slow-moving outer planets the actual computations can be made at longer intervals than four days, and the work still further reduced. But even with the most effective

organisation and with every labour-saving device, adding, multiplying and dividing machines, the work of the *Nautical Almanac* Office is extremely heavy. Eighteen assistant computers were required in the work of preparing the *Almanac* for 1907.

CHAPTER VI

TIME AND ITS DETERMINATION

OUR conception of time has its origin in the regular alternation of day and night. The day is the natural, the fundamental, unit with which we compare and measure other intervals of time, and it has been so used from prehistoric times. Yet the fact that the change from daylight to darkness and from darkness to daylight is caused by a simple rotation of the earth was not clearly recognised and did not become an accepted scientific belief until some fifty years before Columbus discovered America.

In order that the rotation of the earth be a measure of time, it must be perfectly uniform; the day must be of constant length. Now so far as can be determined from observation the rotation is uniform and the day is of the same length now as it was when the shepherds first watched the stars from the plains of Chaldea. From a comparison of the times at which eclipses, transits, and other astronomical phenomena occurred in bygone years, it can be shown that the day has

not changed by so much as the hundredth part of a second since the time of Ptolemy. Yet certain theoretical considerations show that the day must be gradually growing longer. The sun and moon generate tides in the oceans and these tides act as a friction-brake upon the earth and tend to retard its rotation. Meteors fall and are deposited upon the earth's surface. Century after century the earth is thus growing bigger and, as its mass increases, the velocity of its rotation must diminish. These forces, which tend to lengthen the day, are extremely slow in their action and are to a certain extent counteracted by the gradual cooling of the earth and its consequent shrinkage. The movement of any mass toward or away from the axis will of necessity affect the rotation of the earth; the gradual wearing away of mountain ranges and the subsidence of continents, the smoothing out of the surface irregularities—all tend to accelerate the rotation. The effects of all these varying tendencies are extremely small and cannot be computed in even the roughest manner. On the whole, however, the probability is that the day is slowly lengthening and that in the centuries to come the earth will rotate much more slowly than it does at present.

So far the day has been spoken of as if it were perfectly simple and needed no definition. Every one, of course, knows what a day is, or thinks he knows. Yet very few could, offhand, give an exact definition of what a natural, an astronomi-

cal, day really is. In ordinary conversation day is the interval of light, as distinguished from the alternating intervals of darkness, or night; it is the period between the rising and setting of the sun. The true day, on the other hand, the astronomical day, the legal day, the calendar day, includes one period of daylight and one period of darkness; it is the period of time in which the earth rotates once on its axis. As the earth rotates, the meridian of any place, that imaginary line which passes from the north through the zenith to the south, sweeps across the heavens from west to east, passing successively through star after star, through the sun, the moon, and the various planets. If there were in the heavens any absolutely fixed point, then the interval of time between two successive passages of the meridian through that point would be one day and it would exactly measure one complete rotation of the earth. Unfortunately, for simplicity, there are no absolutely fixed points in the heavens; the sun, the moon, and the planets are moving and moving with great rapidity; even the so-called fixed stars are in motion—some are moving one way, some another. If the sun be used to measure the day, one length would be found; if the moon be used, another; and if a "fixed" star, still another. Thus we may have a solar day, a lunar day, and a stellar or sidereal day.

The actual motions of the stars across the heavens are extremely slow; the fastest-moving

star changes its position by only 9″ a year. At this rate it requires over two centuries for the star to move by an amount equal to the apparent diameter of the moon. Yet, slow as this is, it is fast as compared to the motion of the average star. Out of the countless thousands that are visible through our telescopes, less than 100 stars have moved by so much as the moon's diameter since the beginning of the Christian era. Were even the fastest-moving star used to determine the true length of a day, its motion could not introduce an error so great as $\frac{1}{1000}$ of a second. By using many stars the length of the sidereal day, or the actual time of rotation of the earth, can be found with the highest degree of accuracy.

The sidereal day is not convenient for ordinary use. For astronomical observations it is indispensable, but our lives are regulated by the rising and setting of the sun, and for practical everyday use we must utilise the solar day. Now an apparent or true solar day is the interval of time between two successive passages of the centre of the sun across our meridian. On account, however, of the eastward motion of the sun among the stars, this solar day is longer than the sidereal day. If the sun crossed the equator at noon exactly on March 21, then the sun and the vernal equinox would be on the meridian at the same instant, and the solar and sidereal day would begin at the same time. Twenty-four hours later, however, the sun would have moved nearly a degree

to the eastward, and as the rotation of the earth carries the meridian from the west towards the east, the meridian would pass through the vernal equinox before reaching the sun. On March 22, the vernal equinox would culminate nearly four minutes before the sun; the sidereal day, from March 21 to March 22, would be shorter than the solar day by this amount. If now this eastward motion of the sun took place on the celestial equator, and if it were perfectly uniform, then each succeeding solar day would be of the same length; each some four minutes longer than a sidereal day. But, as was noted in a former chapter, this motion of the sun among the stars is neither uniform, nor does it take place on the celestial equator. In December the sun moves faster than it does in June: in twenty-four hours in December the sun changes its place among the stars by some $61' \ 10''$, while in the early part of July it moves in the same length of time only $57' \ 12''$, thus showing a variation of $6\frac{1}{2}\%$ in speed. Again in June and December this motion of the sun is practically parallel to the equator, the whole of the motion being eastward; while in March and September the sun's path crosses the equator at an angle of more than $23°$, and at these dates, therefore, only a part of the actual motion is toward the east. Hence, although the sun may actually be moving faster in September than in June, yet its eastward motion is more rapid in the latter month.

As it is the eastward motion alone which determines the length of the solar day, the days are actually longer in December than in midsummer or in autumn. The shortest day of the year is September 15, when the interval from noon to noon is some 51 seconds shorter than December 24.

The real solar day thus varies, no two successive days being of exactly the same length. It is, therefore, useless as a measure of time. To overcome this difficulty a mean, or average, solar day is used; a day whose length is the average of all the real solar days in one year. This, the mean solar day, is the day of ordinary, business life, the day by which our clocks are regulated. It furnishes an invariable unit of time and is now used for all purposes, except in certain astronomical observations, where sidereal time is employed.

On December 24, mean and apparent solar times coincide: on that day of each year the sun is on the meridian at twelve o'clock mean time, —sun-dial and clock time are the same. At this time the sun is moving more rapidly than on the average, and the true solar day is, therefore, a little longer than the average or mean day. Hence on December 25, it will be mean noon before it is apparent or true noon. The sun will not reach the meridian until our clocks register a few seconds past twelve. As, at this season of the year, each true solar day is about 30 seconds longer than the average, each succeeding true, or

sun, noon will fall some 30 seconds later than the previous. This will go on until by the middle of February noon by the sun is 14½ minutes late by the clock. At this date the true and the solar days are of exactly the same length, but after this date the solar day becomes shorter than the average, and the fourteen minutes difference between the two noons will gradually diminish until April 15, when apparent and clock times again coincide.

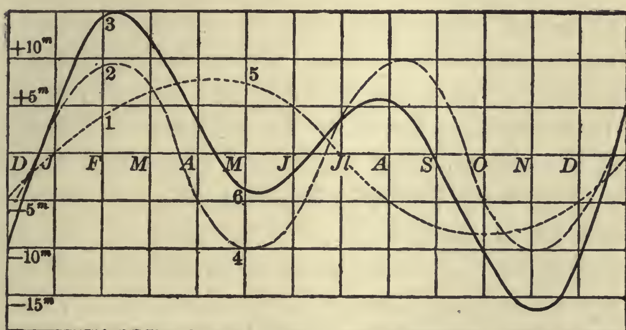

Fig. 7. The Equation of Time

This difference between the apparent or sun time and the mean or clock time is called the "equation of time." It is zero four times a year, namely, on April 15, June 14, Sept. 1, and December 24. On February 11, as has been seen, the sun time is late by nearly 14½ minutes, and again on July 26 the sun time is late by nearly 6¼ minutes; but in May and in November sun time is fast. On December 2, the sun reaches the

meridian nearly $16\frac{1}{3}$ minutes before mean noon. Thus the time as shown by a sun-dial, or as determined by the precise observations with a sextant, will never agree with clock time, and may differ from clock time by over a quarter of an hour. The amount of this difference for any date, or what is the same thing, the value of the equation of time, can be found from the known motions of the sun, and such values are calculated and given for each day in the *Nautical Almanac.*

In speaking of clock time in the above paragraphs the mean solar time of the place is to be understood. Our ordinary clocks and watches no longer run on this time, but keep to a so-called "standard time." In reality each locality has a mean time of its own. When it is mean noon in New York, it is already $11^m\ 46^s$ past noon in Boston, while it still lacks $12^m\ 22^s$ of being noon in Washington. When travel was by stage-coach and it took three or four days to make the trip between Boston and New York, this made little inconvenience. But with thousands of people travelling back and forth each day, and with trains arriving and departing on the minute, it would be extremely inconvenient to have to change our watches upon arrival in a different city. To overcome the confusion resulting from such a diversity of times, all the cities and towns within a certain definite locality now keep a uniform or "standard" time. All the cities along the Atlantic seaboard keep what is called "East-

ern standard time"; this is the mean time of the 75th meridian west of Greenwich, England, and is, therefore, exactly five hours slow on London time. Cities in the middle West keep "Central time," which is one hour slow on Eastern time, or six hours slow on London time.

The Measurement of Time

Among the peoples of olden time the solar day was divided into two periods of twelve hours each and this division has persisted until the present time. The two periods, which now begin at noon and midnight, began, in the times of the early Greeks, at sunrise and at sunset. They divided the period of daylight into twelve parts, and the period of darkness into twelve similar hours. As these two, day and night, are of different lengths, an hour in the daytime differed from an hour at night. Even more confusing, to our modern conceptions, the length of the hour varied with the seasons of the year. If such were the practice to-day, a daylight New York hour in summer would be 75 minutes long, while an hour in midwinter would consist of but 46 minutes.

These hours were measured by sun-dials and by clepsydræ, or water-clocks. In its earliest form the sun-dial consisted of a gnomon, or vertical shaft, the shadow of which marked out the hours. Noon fell at the end of the sixth hour, when the sun was on the meridian and the shadow

the shortest. In later forms of the sun-dial, the gnomon was placed in a hollow basin, the shadow being received on the inner curved surface. On this surface the hours were marked off.

The first public sun-dial in Rome seems to have been brought from Sicily about the year 263 B.C. It was erected near the Rostra and by it the hours of the Romans were indicated, but, like all innovations, it did not at first receive universal approval. In a comedy written nearly half a century later a slave complains of the hardship forced upon him by the introduction of sun-dials; complains that he is forced to wait upon the sun for his meals, while when he was a boy he could eat when he was hungry.

As sun-dials cannot be used at night or during cloudy weather, some other method of noting the passage of the hours is necessary. At very early times this necessity was felt and it led to the invention of the water-clock. They were used by the Chaldeans, they were used in times of the remotest antiquity in China, in India, and in Egypt. These old water-clocks were identical in principle with the hour-glass of to-day, using water instead of sand. A vessel with a small hole near the bottom was filled with water, and as the water flowed out of the orifice it was caught in a second vessel or vase and there measured. Hipparchus, who lived nearly 200 years before Christ, speaks of measuring the diameter of the sun by

means of such a clock; and the Chaldeans, centuries before that date, used clepsydræ not only to mark the hours of the night, but to locate and measure the positions of the signs of the zodiac in the heavens.

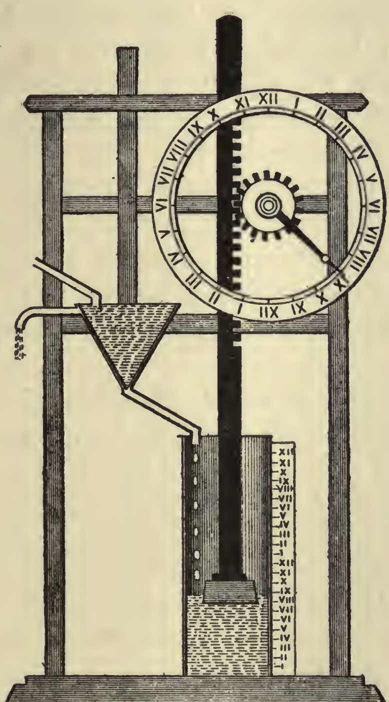

Fig. 8. Antique Form of Clepsydra

Modified forms of water-clocks were used for centuries, but even these were costly and rare. A clock was a present which one king gave to another, and few monasteries were rich enough

to possess one. In every monastery a monk was assigned the duty of watching the stars and the moon, noting the burning of the wax candles, and of calling the hours. In an eleventh-century treatise on the duty of monks a Cardinal thus exhorts the so-called "significator horarum": "Let him be always intent on his duty, and never relax his observation of the revolving sphere, the movement of the revolving sphere, and the lapse of time. Moreover, let him acquire a habit of singing psalms, if he wish to possess a faculty of distinguishing the hours; for whenever the sun or stars are obscured by clouds, the quantity of psalms which he has sung will furnish him with a sort of clock for measuring the time."

The earliest mention of a clock moved by weights and wheels seems to be of one made in Madgeburg about the year 996, but such clocks did not become practicable until the twelfth and thirteenth centuries. One of the earliest clocks of which we have definite knowledge was made by Henry Vick in the year 1370 for King Charles V. of France. With the exception of the balance wheel, or regulator, this clock contained all the essential elements of a modern timepiece. It was driven by a weight and indicated the hours by means of a large hand passing in front of a face or dial divided into twelve parts. The train of toothed wheels or gears, by which the motion of the falling weight was communicated to the hand,

was almost identical with that now found in simple clocks.

The regulating device in the clock of Vick was the forerunner of the balance wheel of the modern watch and chronometer. Every mechanical timepiece must contain some device which oscillates or vibrates back and forth in equal intervals of time, and which, at each oscillation, checks or regulates the movement of trains of toothed wheels. In Vick's clock this oscillating balance was a long horizontal arm or bar supported at its middle point by a twisted cord. A rod so hung will vibrate back and forth in sensibly equal periods of time; the length of the period depending upon the length and weight of the rod. In order to regulate the period of the swing, Vick put weights on the ends of the bar, and by moving these out or in towards the centre the swing could be adjusted to one second.

While this device was extremely crude yet it contained the elements of the balance wheel of the modern watch. The straight rod has been replaced by a wheel and the twisted cord by a fine steel spring.

The pendulum as a measurer of time was invented by Galileo, who, however, used it only as a physician's instrument. He made a very short pendulum, which could be held in one hand and its beat compared with the pulse of a patient. Not until nearly fifty years later, in 1657, was the pendulum successfully applied to

a clock by Huyghens. The introduction of this principle fixed the basis of the modern astronomical clock, and from this time on the development has been along lines then established.

It was not long, however, before a serious difficulty confronted the clockmakers. Clocks ran faster in winter than in summer; the rate of the clock depended upon the average temperature. The explanation is perfectly simple; heat expands all metals, and in summer the pendulum is longer than in winter, and the longer the pendulum the more slowly it swings. An increase of only ten degrees in the average temperature will cause a pendulum to swing so much more slowly that a clock governed by it will lose three seconds a day. This difficulty was overcome by Harrison, who invented the compensated or gridiron pendulum, which is still used in parlour and ornamental clocks. This pendulum consists of a number of steel and brass rods, so arranged that expansions of the different metals counteract each other and the centre of the bob always remains at the same distance from the point of support.

Nowadays, zinc instead of brass is often used in combination with steel, and such pendulums give very good results. Probably a still better form of compensated pendulum, and one more frequently used where great accuracy is required, is that invented by Graham, the so-called mercurial pendulum. In this the bob consists of a

glass or steel jar, two or three inches in diameter and about eight inches high, filled with some forty or fifty pounds of mercury. When the temperature rises the rod lengthens and the jar drops a little, but at the same time the mercury in the jar expands, rises, and lifts the centre of gravity of the mercury. By adjusting the amount of mercury in the jar, the compensation can be made almost perfect; the centre of gravity of the bob or jar remaining always at a constant distance from the point of support.

The daily fluctuations in the pressure of the atmosphere also affect the rate of a clock: a one-inch rise of the barometer will make a clock lose about one third of a second per day. The barometric changes are small, however, and their effect upon a clock is usually neglected. But for extreme accuracy, the clock may be sealed in an air-tight case, or special mechanisms used to counteract the effect of the varying barometer.

The Determination of Time

No matter how carefully made and how accurately it may run, a clock cannot show the correct time unless it be correctly started. In order to set it going we must know the time. Now the ultimate measurer of time is the rotation of the earth, and the only way that this rotation can be measured is by means of the heavenly bodies,—the sun and the stars. The determination of

the time thus of necessity requires astronomical observations.

We have seen that the length of a dayis defined as the interval between two successive returns of a heavenly body to the meridian. A sidereal day is the interval between two successive transits of a star, a solar day between two transits of the sun. Now if we have any means of noting the exact instant when a star or the sun is on the meridian, we at once know the time of noon, sidereal or solar.

The simplest and oldest instrument for noting the time of noon, is the sun-dial. In its simplest form this is a gnomon, or upright pointed stick, which casts a shadow on a level surface. Through the foot of the stick is drawn a north and south line. When the sun's shadow falls on this line it is true solar noon. By adding or subtracting, as the case may be, the equation of time, the instant of mean or clock noon may be readily found. Of course the sun-dial is not an accurate instrument; the shadow is more or less indistinct and by it the time can only be determined in a rough, approximate manner.

The same principle as that of the sun-dial, however, is used in the most accurate method of determining time known to astronomy. The "transit instrument" is but a vastly improved and perfected sun-dial. It is merely an instrument for noting the time that the sun or a star crosses our meridian.

The pointed stick of the sun-dial, which casts

an ill-defined and irregular shadow, is replaced by a carefully shaped and ground lens which forms a clear and distinct image of the sun; the

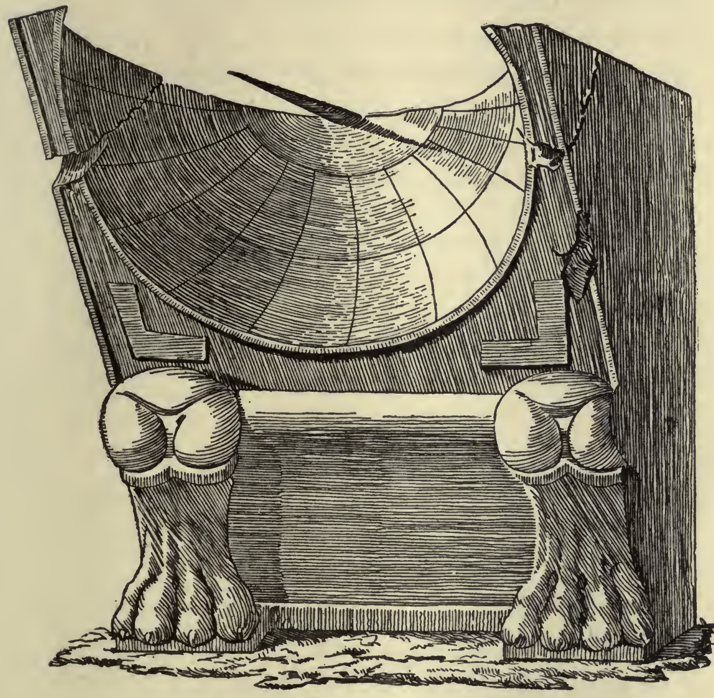

Fig. 9. Old Italian Sun-dial

roughly drawn north and south line of the dial is replaced in the transit instrument by the finest of silken threads, that spun by a spider. Now if such a lens and spider line be mounted in a rigid tube, forming a telescope, and further if the tube be mounted so as to revolve about an axis

at right angles to the line of sight, the completed instrument has all the essentials of a transit instrument. The whole forms a cross, the telescope tube being the longer arm, the pivots about which it revolves the shorter. If these pivots rest in firm sockets, or Y's, placed exactly east and west, so that the axis of revolution is a horizontal line, then, when the telescope revolves, the line of sight will always describe the meridian, and any body whose image falls on the spider line will, at that moment, be on the meridian. With such an instrument in perfect adjustment the rating of a clock, or the finding of the time, is a very simple matter, whether sidereal or solar time be used.

For Sidereal Time

At the instant any star is on the meridian the sidereal time is equal to the star's right ascension.

Except for very small changes due to precession, nutation, and proper motion, a star's position in the heavens is constant; its α changes very slowly. The positions of a large number of the brighter stars are given in the *Ephemeris* or *Nautical Almanac* for each year and thus all the observer has to do is to note on the clock the time of the star's passage. The difference between this time and the right ascension, from the Ephemeris, is the error of the clock. To put it in the form of an equation,

$$\Delta T = \alpha - T$$

where α is the right ascension and T is the observed clock time.

For Solar Time

The instant that the centre of the sun is on the meridian is true, or apparent, solar noon, and this differs from mean or clock noon by the equation of time. On January 21st, the equation of time is given by the almanac as $11^m\ 13^s$ to be added to apparent time, therefore when the sun is on the meridian the clock should read $11^m\ 13^s$ past noon. If the clock reads thus it is correct, if anything different it is in error, and the "clock error" is the difference between what it actually does read and this $11^m\ 13^s$ past noon.

While this is simplicity itself, it is really used but seldom, for the sun is a poor body to observe. Its disc is quite large and it is impossible to tell when its centre is exactly on the thread. The two edges or limbs may be observed and the time half way between them taken as the passage of the centre. But even the edges of the sun cannot be observed accurately, and again the sun heats the instrument, distorts it, and twists the sight-line out of the meridian. For these reasons the sun is seldom observed for time. Stars are used, for they can be observed with great accuracy. The right ascension of the star is the sidereal time of its transit across the meridian. Now by the tables furnished in the *Almanac*, this sidereal

time can be converted into the corresponding mean solar time, and the difference between this and the clock time is the error of the clock.

This is the theory of the simplest and most direct method of determining the time, or correcting a clock. Unfortunately in practice it is not so simple as it seems. No matter how carefully a transit instrument may be made and how accurately adjusted, the line of sight seldom describes the meridian. The meridian of the instrument is not the true meridian and the time we note on the clock is not the time that the star actually crossed the meridian of the place. Even if the instrument could be placed in perfect adjustment, it would not long remain so: the slightest jar, or the unequal heating of various parts of it, would throw it out. So delicate are the larger instruments that the heat from a hand placed on one of the stone piers will appreciably change the level of the instrument and cause the sight-line to describe a curve slightly different from the meridian. While thus the transit instrument can never be exactly in the meridian, yet, from observations made on several stars, some to the north and some to the south of the zenith, its various errors can always be determined, and the effect of these errors on the time of passage of a star can readily be calculated.

It is in this manner that clocks in fixed observatories are regulated. The principal time centre of the country is the Naval Observatory at Washington, where through elaborate observations with

a transit, a standard clock is maintained and its error on 75th meridian time always known. At mean noon of every day an automatic electric signal is given. This signal is distributed over the wires of the Western Union Telegraph Co. from city to city over the entire eastern part of the continent.

For travellers this transit method is not available, for it requires a permanently mounted instrument. Surveying parties on land can and do carry small portable transits with them, instruments which can be quickly set up and adjusted, and used for a few days or weeks in a temporary camp. While not comparable with the accuracy of the fixed instruments of an observatory, yet such portable instruments give the time with a high degree of precision. At sea, however, the transit method fails utterly. With the rolling and pitching of the vessel, no instrument can be maintained in a fixed position; the meridian cannot be determined. One instrument only can be used—the sextant.

This is an instrument that can be held in the hand and with which the angle between any two bodies can be measured. At sea it is used primarily to measure the height of the sun above the horizon.

Now the sun rises in the east, crosses the heavens diagonally towards the south, rising higher and higher until at noon, when it crosses the meridian, it is at its greatest altitude. From this instant it

declines towards the west, getting ever lower and lower. Now the instant of greatest altitude is also (in the case of a star, but not absolutely in the case of the sun) the instant of meridian passage. The altitude of any body from the sea horizon can be easily measured and, at first glance, it would seem, therefore, as though by watching for the time of greatest altitude the instant of meridian passage could easily be determined. This, however, is not the case, for as the sun nears the meridian, it changes its altitude more and more slowly, until finally as it crosses the meridian it is actually moving for the moment parallel to the horizon. With the sextant it is impossible to determine the instant of greatest altitude, or noon, within a minute or more. It is absolutely impossible to determine the time in this way with any degree of accuracy.

But for every altitude of the sun, at a given place each day there is a corresponding definite instant of time. At ten o'clock this morning the sun was so many degrees, minutes, and seconds above our horizon, at ten thirty it was higher, and at eleven o'clock still higher. If the time be known the altitude can be found; if the altitude be known the time can be found. Unfortunately, however, the relation between the two is not a simple one; it involves the use of trigonometry and the solution of a spherical triangle. This triangle and the elements of the problem are shown on the accompanying figure, which represents the visible

hemisphere at any time, the earth being supposed at the centre. The horizontal circle SHN repre-

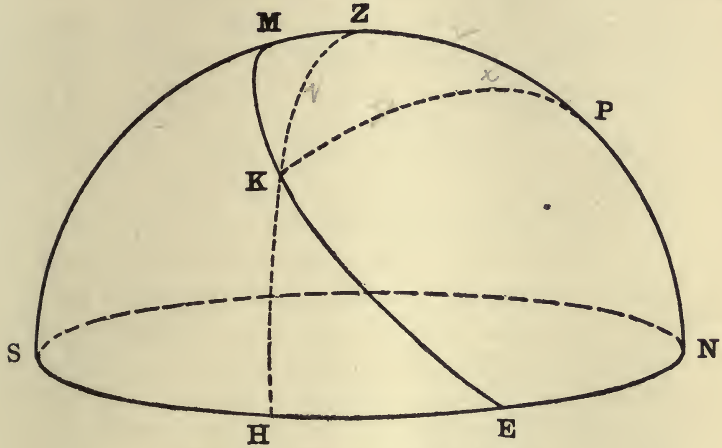

Fig. 10. Time by a Single Altitude

sents the observer's horizon, N being the north point and S the south point, the arc SZPN will be the meridian, Z being the zenith and P the north pole of the heavens. Owing to the rotation of the earth the sun appears to describe a small circle about the pole P, rising above the horizon at E, and crossing the meridian at M. At any given instant the sun will be found at K, its altitude or distance above the horizon being HK. Now the angle MPK, which changes as the sun rises, is the "hour angle," and it directly measures the number of hours that it will take the sun to reach the meridian. When the sun is on the meridian this angle is zero; at six o'clock in the

morning, or at six o'clock in the evening it is 90° or 6^h, at midnight it is 180° or 12^h.

Now the figure shows very clearly that as the sun rises, the hour angle becomes smaller while the altitude increases, but it is also evident that there is no simple relation between the two. The necessary relation between the two is furnished by the spherical triangle ZPK. The three sides of this triangle need but little explanation: ZK is the zenith distance (z) of the sun, or is 90° minus the altitude; PK is the polar distance, or 90° minus the declination (D) of the sun; PZ is the polar distance of the zenith, or 90° minus the latitude (L) of the place of observation. The angle ZPK has already been noted as the hour angle and is usually denoted by t. Now spherical trigonometry furnishes a definite relation between these four quantities, and that relation is expressed by the formula

$$\cos z = \sin L \sin D + \cos L \cos D \cos t.$$

and this formula is the fundamental formula upon which the whole of navigation rests. It is used, as will be seen later, not only for determining time, but also for finding latitude and longitude. It is the one essential astronomical formula for the navigator to know and to understand.

It involves four quantities; if any three of them are known, the fourth can be found from it by calculation. It can be transformed and put into better or more convenient shape for numerical

calculation, the exact shape in which it is used depending upon the unknown quantity sought. In the case now under discussion, we are supposed to know our latitude and the declination of the sun, and to have measured the altitude. The unknown quantity, the quantity which we wish to determine from this formula is t, the time. Solving the formula for the value of t, we find:

$$\sin^2 \tfrac{1}{2}t = \frac{\sin\tfrac{1}{2}[z+(L-D)]\ \sin\tfrac{1}{2}[z-(L-D)]}{\cos L \cos D}$$

which can be solved readily by means of the logarithmic tables of sines and cosines.

If it is desired to use the altitude, h, directly in the formula instead of the zenith distance, it can readily be done. But at the same time it will be advantageous to use the polar distance ($90°—D$) instead of the declination. When these substitutions are made the formula becomes

$$\sin^2 \tfrac{1}{2}t = \frac{\cos s \sin (S-h)}{\cos L \sin P}$$

where P is the polar distance and

$$S = \tfrac{1}{2}\ (L+P+h).$$

Some prefer one shape of the formula, some the other. The result is the same, of course, whichever form is used.

This is the most important formula in the whole of navigation and is the only one that presents any difficulties. To a practised computer it is very simple and easy of application, but to the

navigator it is rather complicated and many attempts therefore have been made to simplify it.

If ordinary logarithmic tables are used the value of $\frac{1}{2}t$, or one half the hour angle of the sun or star, is found. The formula gives this angle in arc; in degrees, minutes, and seconds, so in order to find the hour angle itself we must multiply by two (2) and divide by fifteen (15). Bowditch in his tables of sines and cosines, as arranged for the navigator, gives in separate columns the values of the angle in degrees and minutes, and also the value of twice the angle when expressed in hours and minutes of time. When such a table is used the work is considerably shortened for the value of t, in hours, can at once be found from the logarithm of the sine of $\frac{1}{2}t$. Many of English nautical works contain tables of "Haversines" (half-versine), and these still further shorten the work. The haversine of t is equal to $\sin^2\frac{1}{2}t$, and, therefore, with a table of haversines, the value of t can be at once found from the logarithm of the $\sin^2\frac{1}{2}t$. In Inman's Tables, the angle, t, is expressed both in arc and in time, so that the work is reduced to a minimum.

When the sun is the object observed this formula, no matter what form it may be put into for convenience of computation, gives the value of t, the hour angle of the sun, or the apparent solar time at the moment the observation was made. By applying to this the equation of time, as taken

from the *Nautical Almanac*, the mean solar, or local clock, time is at once obtained.

When a star is observed this formula gives the hour angle of the star. If the star be west of the meridian, this hour angle added to the star's right ascension gives the sidereal time; if the star be east of the meridian, the sidereal time is found by subtracting the hour angle from the right ascension. If desired this sidereal time can be converted into mean time, by means of the data in the *Nautical Almanac*. Some care should be exercised in selecting a body to be observed and the time at which to make the observation. It is obvious that the best results can be obtained by an observation made at a time when the altitude of the body is changing most rapidly. For a slight error in measuring the altitude will then introduce a small error only in the time, but when the altitude is changing very slowly it is impossible to determine at exactly what moment the body has a definite altitude. The nearer a body is to the equator, the faster it moves. The Pole Star changes its altitude by only 2° in twenty-four hours, while a body on the equator rises an equal amount in about eight minutes. Hence for finding the time, stars of small declinations should be used; the bright stars of the northern heavens are useless for this purpose. As the sun never departs more than 23° from the equator, it is always available and may be used to advantage.

Again with any given body, the sun or a favour-

ably situated star, the altitude will change most rapidly when the body is on the prime vertical. When the body is near the meridian, it is momentarily travelling nearly parallel to the horizon and its altitude changes very slowly. Hence for determining the time with precision, an observation should always be made when the body bears as near east or west as possible. It is azimuth, not hour angle, which determines the proper time for making an observation.

NOTES AND PRACTICAL APPLICATIONS

1.	Number of hours in a day	24
	" " minutes " "	1440
	" " seconds " "	86400

Longest day of the year, December 24, $24^h\ 0^m\ 30^s.2$

Shortest day of the year, September 15, $23^h\ 59^m\ 38^s.7$

Equation of time—Maxima—	Feb. 11	$+\ 14^m\ 32^s$
	July 26	$+\ 6^m\ 12^s$
Minima—	May 14	$-\ 3^m\ 55^s$
	Nov. 2	$-\ 16^m\ 18^s$
Zero—	Apl. 15	
	June 14	
	Sept. 1	
	Dec. 24	

Astronomical, or almanac, day begins at *noon.*

Civil day begins at *midnight.*

Civil day begins 12 hours before astronomical day.

Day begins in longitude 180° east or west from Greenwich.

One sidereal day equals 0.997,269 solar days.

One solar day equals 1.002,738 sidereal days.

24 hours sidereal time equals $23^h\ 56^m\ 4^s.091$ solar time.

24 hours solar time equals $24^h\ 3^m\ 56^s.555$ sidereal time.

Acceleration of sidereal time in one solar minute, $0^s.164$

Acceleration of sidereal time in one solar hour, $9^s.8565$

Acceleration of sidereal time in one solar day, $3^m\ 56^s.555$

2. *Conversion of Time:*

(a) Local time (solar or sidereal) into Greenwich time.

To find the Greenwich time corresponding to the local time (apparent, or mean solar or sidereal) of any meridian, *add* the longitude of the place expressed in hours to the given local time. Longitude should always be reckoned positive when west of Greenwich, negative when east.

Example:

Washington local apparent time	$10^h\ 41^m\ 10^s.00$
Longitude	$5\ \ 8\ \ 12\ .04$
Greenwich apparent time	$15^h\ 49^m\ 22^s.04$

(b) Apparent solar time into mean solar and the reverse.

The two kinds of time differ by the equation of time, and the value of the equation is given in the *Nautical Almanac* for every day of the year. Find the equation from the almanac and add or subtract it as directed.

Example—October 19, 1908:

Greenwich apparent time	$15^h\ 49^m\ 22^s.04$
Equation of time, *add* to mean	$15\ \ 3\ .24$
Mean time	$15^h\ 34\ \ 18^s.80$

(c) Mean solar into sidereal time.

Convert the Greenwich mean solar time into sidereal hours, minutes, and seconds by adding the sidereal acceleration, or by Table III, *N.A.*, and the result is the number of sidereal hours between Greenwich mean noon and the moment of observation. To find the Greenwich sidereal time, add to this interval the sidereal time at Greenwich mean noon as taken from the *Nautical Almanac*. If the sidereal time at the ship be required, subtract the longitude.

Example—October 19, 1908:

Greenwich mean time	$15^h\ 34^m\ 18^s.80$
Sidereal acceleration (Table III, *Nautical Almanac*)	+ 2 33 .48
Sidereal time at Gr. mean noon (*Nautical Almanac*)	13 50 13 .64
Greenwich sidereal time	$5^h\ 27^m\ 5^s.92$
Longitude	5 8 12 .04
Washington sidereal time	$0^h\ 18^m\ 53^s.88$

(d) Sidereal into mean solar time.

From the Greenwich sidereal time (+24 hours, if necessary) subtract the sidereal time at Greenwich mean noon, as taken from the *Nautical Almanac*. Reduce the resulting sidereal interval to the corresponding mean time interval by subtracting the sidereal acceleration, or by Table II, *N.A.* The result is the Greenwich mean solar time, and this may be changed to ship time by subtracting the longitude.

Example—Oct. 19, 1908:

Washington sidereal time	0^h	18^m	$53^s.88$
Longitude	5	8	12 .04
Greenwich sid. time ($+24^h$)	29	26	65 .92
Sidereal time at Gr. m. n.	13	50	13 .64
Sidereal interval	15	36	52 .28
Reduction Table II, *N.A.*	—	2	33 .48
Greenwich mean time	15	34	18 .80
Longitude	5	8	12 .04
Washington mean time	10	26	6 .76
Equation of time	+	15	3 .24
Washington local app. time	10^h	41^m	$10^s.00$

3. *Determination of Time:*

(a) Formulas for finding time from a single altitude of sun or star.

From measured altitude corrected for refraction, dip, and semi-diameter, if necessary, find hour angle of the body by formula,

$$\sin^2 \tfrac{1}{2}\, t = \frac{\cos S \sin (S-h)}{\cos L \sin P}$$

where,

$$S = \tfrac{1}{2}\,(L + P + h)$$

and

L = latitude of place.
P = polar distance of body.
h = measured altitude.

Thence,

1st for sun.

The value of t as above is the local apparent, or ship's, time.

To find the local mean time, apply the equation of time, as given in the *Nautical Almanac.*

To find Greenwich mean time, add the longitude to the local mean time.

2nd for planet or star.

To or from the right ascension of the body add (if west) or subtract (if east) the hour angle as above, and the result is the local, or ship's, sidereal time.

To find local or Greenwich mean time when the longitude is known, convert this sidereal time as in section 2.

To find the local time when the Greenwich time is known, find, for the time of observation, the right ascension of the sun. The difference between this and the sidereal time is the hour angle of the sun, or the local apparent time.

(b) Practical examples.

On May 27th in latitude 40° 44′ N. four altitudes of the sun's lower limb were observed in quick succession and the times noted on chronometer regulated to Greenwich mean time: index correction + 1′ 10″; height of eye, 10 feet. Required the local mean time.

	Observed Altitudes	*Chronometer Times*
	43° 0′	$8^h\ 6^m\ 10^s$
	42 50	7 0
	42 40	7 52
	42 30	8 46
Mean	42° 45′	$8^h\ 7^m\ 27^s$
Index	+ 1 10″	
Dip	— 3 6	
Table IV	+ 15 2	
Table V	— 11	
h	42° 57′ 55″	
L	40 44	
P	68 39 56	
	2)152° 21′ 51″	
S	76° 10′ 55″	
$S-h$	33 13 0	

Declination

At G. m. noon,	21° 16′ 39″.2 N.
Corr. for $8^h.12$	+ 3 24 .9
Decl.	21° 20′ 4″.1 N.

Equation of Time

At G. m. noon,	$3^m\ 6^s.08$
Corr. for $8^h.12$	— 2 .29
Eq. of time	$3^m\ 3^s.8$
	(*add* to mean)

cos S	9.37811
sin $(S-h)$	9.73863
sec L	0.12047
cosec P	0.03083
Haversine,	9.26804

Apparent time	$3^h\ 24^m\ 1^s$
Equation of time	— 3 3 .8
Mean time	$3^h\ 20^m\ 57^s.2$

The local mean time required is, therefore, $3\ 20^m\ 57^s.2$

If a table of haversines is not at hand, the work of finding the hour angle would be as follows:

$\cos S$	9.37811
$\sin (S-h)$	9.73863
$\sec L$	0.12047
$\operatorname{cosec} P$	0.03083
$\sin^2 \frac{1}{2} t$	9.26804
$\sin \frac{1}{2} t$	9.63402
$\frac{1}{2} t$	$25^\circ\ 30'.15$
t	$51^\circ\ 0'.30$

Apparent time $3^h\ 24^m\ 1^s.2$

And this agrees very closely with that found above; but the work is considerably longer.

On June 19th in latitude $40^\circ\ 20'$, the true altitude of Vega was $41^\circ\ 40'\ 26''$, at $13^h\ 22^m\ 10^s$ Greenwich mean time. Required the local mean time.

h	$41^\circ\ 40'\ 26''$	Vega, R. A.	$18^h\ 33^m\ 49^s.4$
L	40 20	Decl.	$38^\circ\ 41'\ 51''$.
P	51 18 9		
	2)$133^\circ\ 18'\ 35''$	Sun's R.A. Gr.m.n.	$5^h\ 50^m\ 14^s.4$
S	$66^\circ\ 39'\ 18''$	Corr. for $13^h.37$	2 19 .4
$S-h$	24 58 52	Sun's R.A. at obs.	$5^h\ 52^m\ 34^s$
$\cos S$	9.59799	R.A. of meridian	14 17 35
$\sin (S-h)$	9.62564		
$\sec L$	0.11788	Hour angle of sun	$8^h\ 25^m\ 1^s$
$\operatorname{cosec} P$	0.10765	Equation of time	+ 1 8
Haversine	9.44916	Local mean time	$8^h\ 26^m\ 9^s$
Hour angle	$4^h\ 16^m\ 14^s.6$		
Right ascen.	18 33 49 .4		
Sidereal time	$14^h\ 17^m\ 35^s$		

The required local mean time is $8^h\ 26^m\ 9.^s$

If the longitude, 74° 0′, were known instead of the Greenwich time, then the local mean time would be found as follows:

Sidereal time,	$14^h\ 17^m\ 35^s$
Longitude,	4 56
Gr. sider. time	$19^h\ 13^m\ 35^s$
Sidereal time, (Gr. mean noon)	5 49 14
Sidereal interval	$13^h\ 24^m\ 21^s$
Reduc. (Table I)	− 2 11 .8
Gr. mean time	$13^h\ 22^m\ 9^s.2$
Longitude,	4 56
Local mean t.	$8^h\ 26^m\ 9^s.2$

CHAPTER VII

FINDING ONE'S POSITION AT SEA

ASTRONOMICAL observations furnish the only means of finding one's position at sea or upon unknown and unmapped portions of the land. For short coasting trips from port to port, the compass and the log may suffice. But the oceans are full of streams and currents, whose directions and speeds change from week to week and from day to day and dead reckoning is not to be relied on. Cases are on record where vessels have been carried eighty miles off their courses in a single twenty-four hours. In modern steel and iron vessels the compass is subject to deviations, and unless carefully adjusted and tested from time to time it is not to be trusted, and too implicit a reliance upon the compass has been the cause of many a shipwreck.

The principles, upon which is based the art of finding one's position at sea, are very simple and can be explained without the use of any mathematics. Suppose one is travelling across a trackless plain or desert, without compass or guide,

and finds scattered over the plain at irregular but frequent intervals posts which former travellers had set up as guides. Suppose that one or more of these posts was always in sight, and that each post was distinctly marked with the exact position and with the directions and distances to various points on the boundaries of the plain. Would travel across such a staked plain be at all difficult? By simply reading the inscription on the nearest post one could instantly find out one's exact position, and travel directly towards his destination. Now this case of a staked plain is not imaginary, it exactly describes the case of the traveller by sea or by land. Previous travellers, or astronomers, have placed or rather marked and labelled guide posts, not indeed in the sea itself, for there they would be obstructions to navigation, but in the heavens where they are visible to every one and where every one may read their plain directions.

The heavens appear like a great hollow globe surrounding the earth: the sun, the planets, and the stars, each marks a definite point on this celestial sphere. The two globes, the earth and the heavens, correspond to one another, point to point and line to line. To the north pole of the earth corresponds the north pole of the heavens; to the equator of the earth, the celestial equator; the north pole of the heavens, closely marked by the Pole Star, is directly over the north pole of the earth; the celestial equator is directly

over the earth's equator. When Peary reached the north pole, he must have had the north celestial pole directly over head, or in the zenith. Now when we want to locate the position of a town or city on the earth, we give its latitude and longitude, its distance north or south of the equator, and its distance east or west from Greenwich. Similarly in the heavens the position of each star is known by its distance from the equator, its declination as it is called, and by its distance from the celestial Greenwich, or its right ascension. If now the celestial first meridian were directly overhead at London, then the latitude and longitude of any and of every point on the earth's surface would correspond exactly to the declination and right ascension [1] of that point of the heavens which was in its zenith. Directly over New York would be found a faint star barely visible to the sharpest eye, whilst the brilliant star Vega would mark the place of the little town of Redknob in West Virginia.

Now on the staked plain it is not necessary for the traveller actually to touch one of the guide posts in order to find out his position. He may note the sign from afar, and if he knows, or can find, his distance from the post he can locate his exact place in desert wastes. So with the celestial guide posts it is not necessary for one of the stars to be exactly in the zenith, provided

[1] Excepting that right ascension is measured to the east, while longitude is measured to the west and east.

its distance from the zenith can be accurately measured. If, for example, Vega be directly over the town of Redknob, and a traveller find Vega to be one degree (1°) south of his zenith, he then knows that he is one degree, or sixty nautical miles north of Redknob, or in the immediate vicinity of Caldwell, Ohio. The traveller will always be just as many miles to the north and west of Redknob, as Vega appears minutes of arc south and east of his zenith; and vice versa, when he is east and south of Redknob, Vega will appear north and west.

This is the whole essential of navigation,—a star of known position in the heavens is selected and its distance from the observer's zenith measured. The ship will then be as many miles from the point on the earth's surface directly under the star, as the star is minutes of arc from the zenith. To determine the ship's direction from the substellar point it is only necessary to accurately note the azimuth of the star. The zenith distance of the star gives distance, the azimuth gives direction. The zenith distance is measured with a sextant, the azimuth may be roughly found by compass, or more accurately determined by calculation.

So far it has been assumed that the heavens appear stationary, that over each point of the earth's surface is a fixed star. This is not so: the earth rotates on its axis once in twenty-four hours and causes all the heavenly bodies apparently to

describe small circles about the pole, rising in the east, passing across the heavens, and setting in the west. Each star thus travels over its own circle of latitude; Vega, for example, will, at some instant of each day, appear directly in the zenith of every point of the earth's surface, whose latitude is the same as Redknob. As has been noted in former chapters, however, this rotation of the earth is perfectly uniform and furnishes our measure of time; clocks, sidereal or solar, are regulated and governed by this rotation. Sidereal clocks indicate noon, or zero hours, when the first meridian of the heavens is directly overhead; one o'clock when this prime meridian has passed one twenty-fourth of the circumference to the westward. Vega, whose right ascension is 18 $33^m.8$, will always cross one's meridian when the clock points to 18^h $33^m.8$. Three hours before this time, or at 15^h $33^m.8$, Vega will be over a place situated three hours, or 45°, east of the meridian for which the clock is set. Thus by merely noting the sidereal time, the exact spot on the earth's surface in the zenith of which Vega appears, is at once known. To find the substellar point, therefore, it is necessary for the observer to know the sidereal time at which the observation is made. He must have a clock or chronometer regulated to some standard meridian, from which the time can be taken.

Exactly similar reasoning applies to the case of the sun. At apparent noon at Greenwich the

sun is in the zenith of that point on the Greenwich meridian whose latitude is equal to the sun's declination. As the earth rotates the sun (except for its variation in declination) passes along a circle of latitude on the earth, and each hour appears on the zenith of a place 15° further to the westward. At five hours' Greenwich apparent time the sun will be in the zenith of a place 75° to the westward of Greenwich. At any instant, therefore, the latitude of the subsolar point is equal to the declination of the sun, and the longitude, reckoned west from Greenwich, is equal to the Greenwich apparent solar time at that instant. To find the subsolar point it is necessary that the observer shall know the Greenwich apparent time; in other words, the possession of a clock (or chronometer) regulated to Greenwich time is absolutely essential.

Now suppose that a traveller, provided with a chronometer and thus knowing the exact Greenwich time, measures with a sextant the distance of the sun from his zenith. Just what information does the zenith distance furnish him? Does it, by itself, fix his exact location on the earth's surface? If this distance be zero, and the sun appears directly in his zenith, then is his exact locality fixed, for he must be at the subsolar point, and this point is determined by the Greenwich time. If, however, the zenith distance be 10°, then the observer's locality is not thereby determined, for there are many places on the

earth's surface at which the sun's altitude would be 80° at the same instant. Observers 10° north, south, east, and west of the subsolar point would all, at the same instant, have the sun 10° from their respective zeniths. All observers on the circumference of a small circle of 10° radius, whose centre is the subsolar point, would thus at the same instant have the sun 10° from their zeniths. The zenith distance alone, therefore, does not determine the observer's exact position; he may be anywhere on the circumference of the circle, whose radius is equal to the zenith distance. The greater the zenith distance, the larger the "circle of position" and the more indefinite the observer's position. Such a circle of position is shown in the following diagram, which is adapted from Lecky's *Wrinkles in Practical Navigation.* It represents a fictitious observation made on March 7th at one o'clock Greenwich apparent time, when the sun's true altitude was found to be 50°. The sun's declination, as taken from the *Nautical Almanac,* was 4° 59′ south, and consequently the subsolar point was situated in the South Atlantic, in longitude 15° W. and latitude 4° 59′ S. With this point as a centre and with the observed zenith distance, 40°, as radius describe a circle. This is the "circle of position," and to any one situated anywhere on this circle, whether on land or on sea, the sun would appear 40° from his zenith, or 50° above his horizon. The observed altitude by itself, therefore, would merely indicate to the

observer that he was at some point on this circle,—neither inside nor outside of it, but on it; and this

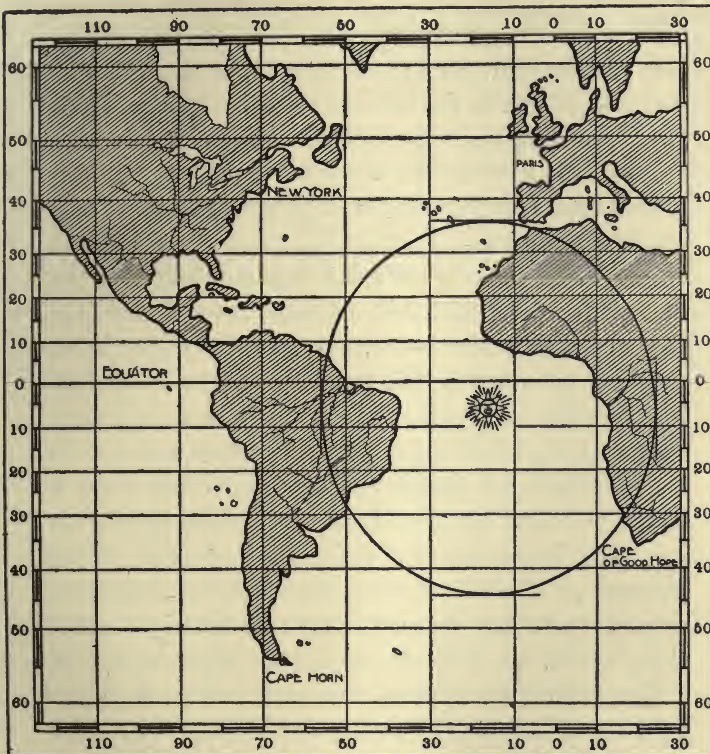

Fig. 11. A Circle of Position

is the sum total that can be derived from the measured altitude. Mathematical calculation is no use, logarithms are of no avail, the measured altitude tells the observer that he is on this circle, but not upon any definite portion of it. To find

his exact position on the circumference of this circle, the navigator must make some other observation, or have some further data to aid him.

Now it is very easy to draw this circle in theory, but rather difficult in practice. On a globe it is simplicity itself, for the circle can be described readily with a pair of dividers. But on a chart it cannot be done in this way, for a chart does not give a true representation of large portions of the earth's surface. Mercator's projection, which is used for charts covering large areas, gives a distorted representation of the earth's surface; the meridians are shown as parallel instead of converging towards the poles, and the circles of latitude are unequally spaced, being much farther apart in high latitudes than at the equator. In latitude 60° an island would appear on such a chart to have an area four times as large as an island of the same actual area at the equator. This necessary distortion of the earth's features causes a similar distortion in the circle of position, which on a Mercator's chart appears not as a circle, but as an irregular oval with its larger axis in a north and south direction. The four cardinal points of the circle can readily be plotted; but the intermediate points can only be found by calculation. The north point of the circle, for example, will have the same longitude as the subsolar point, but will differ in latitude by an amount equal to the zenith distance; the east point will have the same latitude as the subsolar point,

or centre, its longitude, however, being found by subtracting the zenith distance from the longitude of the centre.

The only way the intermediate portions of the circle can be plotted is by finding the latitude and longitude of a number of such points and drawing a smooth curve through them. A glance at our former diagram shows that each circle of latitude cuts the position circle, or oval, in two points and two points only; each meridian of longitude also cuts the circle in two points and two points only. If, therefore, any circle of latitude be assumed, say 30° north, then the longitudes of the two points in which this circle cuts the position circle can be found by calculation. These two points are generally in widely different portions of the earth and can give rise to no confusion in drawing the different portions of the position circle.

Now to find the longitude of a point on the circle of position corresponding to a given latitude, reference must be had to the accompanying diagram. This represents the hemisphere of the earth as viewed from that point over the equator whose longitude is equal to the longitude of the subsolar point. In the figure, S is the subsolar point, and the circle of position appears as an ellipse, which is cut by the 30° circle of latitude in the points O and O′. The difference of longitude between S and O is the arc of the equator intercepted between the meridians passing through

these points. It is measured either by the arc, EE′, or by the angle t at the north pole, between

Fig. 12. Finding the Circle of Position

the meridian SN and ON. But this angle t, is the hour angle of the sun, as seen by an observer in O; it is the local apparent time of the place O. Hence, the problem of plotting the circle of

position, or of finding definite points of the curve, reduces to the familiar problem of finding the local apparent time. And this problem was fully discussed under the head of the "Determination of Time," where it was shown that from the observed zenith distance, together with the declination of the sun and the latitude of the place, the time can readily be found by the aid of a simple trigonometrical formula.

The usual form of this formula is, as is well known,

$$\sin^2 \tfrac{1}{2}\, t = \frac{\sin S \sin (S-h)}{\cos L \sin P}$$

Johnson's form,

$$2 \sin^2 \tfrac{1}{2}\, t = \frac{\cos (L-D) - \cos Z}{\cos L \cos D}$$

when used with his tables is much simpler of application. It is especially useful in the present problem of plotting the circle of position, for in this problem the latitude is not *known*, it is only assumed, and it may therefore be assumed so that $(L-D)$ shall be equal to a whole degree, or to one of the angles for which his table is directly computed. Thus no interpolation is required and the problem is very easily worked.

In actual practice it is not necessary for the navigator or traveller to plot the entire circle of position. Except in very unusual cases he knows his position within half a degree or so, and he only needs, therefore, a short portion of the circle. Now in general the circle is very large,

and any small portion of its circumference may be considered a straight line. Such a straight line is in reality a tangent to the circle and coincides with it for a short distance; and such line is known as a "line of position" or as a "Sumner line," after Capt. Thos. H. Sumner of Boston, who, in December 1837, discovered the basic principle of modern navigation. Perhaps no better explanation of the practical application of the Sumner line can be made than that given by Capt. Sumner himself. In his book, published in 1843, after describing a very stormy passage from Charleston, S.C., to Greenock, when for many days observations were impossible, he continues:

. . . At about 10 A.M. (18th December) an altitude of the sun was observed, and chronometer time noted; but having run so far without any observation, it was evident that the latitude by dead reckoning was liable to error and could not be entirely relied upon.

However, the longitude by chronometer was determined, using the uncertain D.R. latitude, and the ship's position fixed in accordance. A second latitude was then assumed 10′ to the north of the last, and working with this latitude a second position of the ship was obtained; and again a third position by means of a third latitude still 10′ further north.

On pricking off these three positions on the chart it was discovered that the three points were all disposed in a straight line lying E.N.E. and W.S.W., and that when this line was produced in the first named direction, it also passed through the Smalls

light. The conclusion arrived at was, "that the observed altitude must have happened at all three points, at the Smalls light, and at the ship at the same instant of time." The deduction followed that, though the absolute position of the ship was doubtful, yet the true bearing of the Smalls light was certain, provided the chronometer was correct. The ship was therefore kept on her course, E.N.E., and in less than an hour the Smalls light was made bearing E. by N. ½ N. and close aboard. The latitude by D.R. turned out to be 8′ in error.

In this, the first time a Sumner line was used, its essential principles as a "line of position" are clearly stated. The measured altitude of the sun, or star, determines a line upon which the observer must be situated. His exact location on the line is unknown, but the position and direction of the line itself are fully and completely determined. In many cases, especially in approaching a coast, this line of position gives all the information that the navigator needs. The observation may be so timed that the Sumner line shall lie nearly parallel with the coast, and thus it gives accurately the offshore distance. For vessels approaching New York, an observation in the morning gives a Sumner line nearly parallel with the coast line and hence the distance of the ship from land. Again, as in the original example of its use, the Sumner line may intersect the coast and thus give, at once, the true bearing of some important land mark. By steering a

course on, or parallel to, the Sumner line the navigator can be sure of his land fall.

In plotting the original line of position Captain Sumner used three points. He worked through the long trigonometrical formula three times, using three different assumed latitudes. Now it is evident that one third of this work was unnecessary. The line is straight, and two points fix the position of a straight line. To *accurately* locate the line of position it is only necessary to work through the formula twice, using assumed latitudes, differing by half a degree or so. But for the purpose of practical navigation half even of this work can be saved, and the line of position located with a sufficient degree of approximation. The Sumner line is a tangent to the circle of position, and all tangents are at right angles to their respective radii. The Sumner line is, therefore, *at right angles to the sun's true bearing.* If the sun's bearing and one point of the Sumner line be known, the line itself can be drawn, for it must pass through that one point and at right angles to the bearing of the sun. Now, in order to draw the line, it is not necessary to know the bearing of the sun with absolute precision; an error of one degree (1°) in the bearing will introduce an error of only about one half (½) mile at the ends of a Sumner line sixty (60) miles long. This is insignificant and practically negligible. The bearing of the sun can, therefore, be taken with sufficient accuracy from Davis's Azimuth Tables.

The practical method, therefore, of drawing a Sumner line, is to work the sight *once only*, using the latitude by D.R., or if Johnson's Tables be available, the latitude nearest that by D.R. which will make $(L-D)$, a whole degree. Mark the resulting position on the chart. With this same latitude and declination, and with the hour angle found in working the sight, take from Davis's Tables the true bearing of the sun. At right angles to this bearing and through the point marked on the chart, rule the Sumner line. The whole operation is extremely simple, and if Johnson's Tables be used, does not involve more than half a dozen figures. The position of the Sumner line can be computed and it can be plotted on the chart in three or four minutes.

The accurate location of the Sumner line depends directly upon two essentials: the knowledge of the Greenwich time, and an accurate measure of the altitude of the body. For the first of these reliable chronometers are necessary and a large vessel should be provided with at least three, whose errors and rates have been carefully determined. If a single chronometer be used and if it has an unknown error, then the resulting line of position will be pushed east or west from its true location by an amount equal to the chronometer error. A chronometer error of 4^s in noting the time at which the sight was made, will introduce an error of $1'$ in the position of the Sumner line. If through unavoidable circumstances the chro-

nometer cannot be depended upon, and the time is uncertain to 20^s or so, then two parallel Sumner lines may be drawn on the chart, through points differing in longitude by 5′. The ship must then be somewhere on or between these two lines.

Again, if, through a hazy or unsteady horizon, the observed altitude is in error by any amount then the Sumner line will be pushed directly to or from the sun, by an equivalent amount. If the altitude be doubtful by so much as 5′, then two Sumner lines may be drawn on the chart 5′ apart, and the ship will be located somewhere on or between these lines.

The above method of finding the line of position is the one usually adopted, but, while it gives sufficiently accurate results for practical navigation, it is not theoretically the best. Captain Marcq St. Hilaire, of the French navy, has developed, under the name of "new navigation," a method which is theoretically more perfect, and which is equally accurate whether the observation be made on the prime vertical, or near the meridian. Unfortunately, however, the calculations involved in this method are such as to render its general adoption extremely doubtful.

As has been noted the measured altitude determines a definite circle of position, and this circle can be plotted on the chart. In general the position of the ship, as plotted by dead reckoning, will not fall upon the circumference of this circle. Only when the position by dead reckoning agrees

exactly with the true position of the ship, will the circle of position pass through the ship's plotted

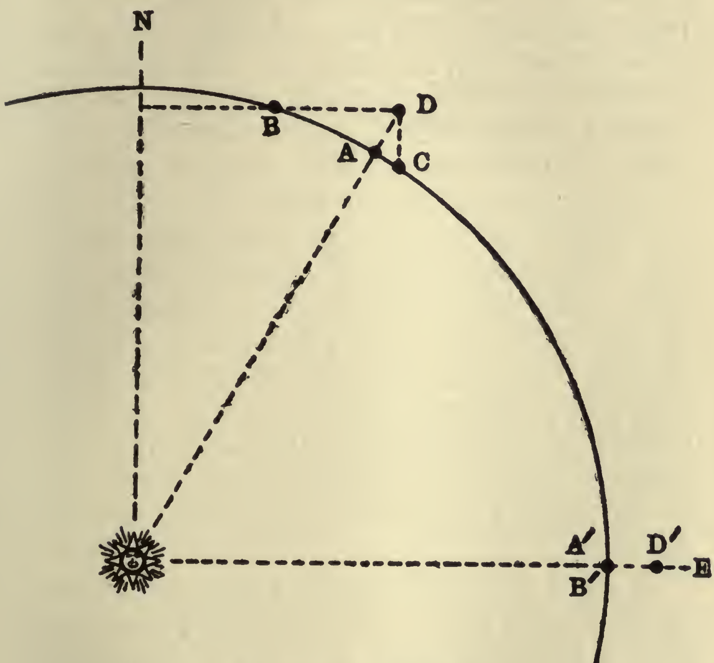

FIG. 13—New Navigation

place on the chart. The ship will usually appear to be within or without the circle, or at D of the accompanying diagram. If now the azimuth of the sun could be accurately measured then the exact position of the ship on the circle could be determined. But this cannot be done, for there are no portable instruments for measuring direction with the requisite degree of precision. In order,

therefore, to pass from the position by dead reckoning to the most probable position of the ship on the circle we must make some assumption; must pass along some line. The most probable assumption is that the ship is on the straight line joining the position by dead reckoning to the subsolar point,—that the ship in reality is at the point A, of the diagram. Other assumptions might be made; either that the latitude by dead reckoning is correct or that the longitude is correct. The first of these would place the ship at B, the second at C. Through any one of these three points a line of position can be drawn tangent to the circle. Now the first assumption, that the ship is at A, or on a line passing through A, is the basis of the method of Marcq St. Hilaire; the second, that the ship is at B, or on a line passing through B, is that of the ordinary method of Sumner lines as now practised.

To apply this method of St. Hilaire the zenith distance of the sun, as it would be seen from the place of the ship by dead reckoning, is computed. In order to do this the hour angle of the sun is calculated from the chronometer time of observation and the D.R. longitude, and then from this hour angle the D.R. latitude, and the declination of the sun, the zenith distance is found by means of the fundamental equation of navigation as given in the chapter on Time. The difference between this computed zenith distance and the observed zenith distance is the distance D A of

our diagram. The azimuth of the sun can be found from Azimuth Tables, or if extreme precision is required it can be calculated from the assumed quantities at the same time that the zenith distance is computed. With this azimuth and the distance D A, the point A can be plotted on the chart, and the line of position drawn through it at right angles to the direction of the sun.

The advantages of the new navigation are great; the method may be used with observations made at any time, with equally good and consistent results. This is not true of the ordinary method, which fails when the observation is made near the meridian. The two methods give identically the same result when the observation is made on the prime vertical; at all other points Hilaire's method is the better. The disadvantages of the new method, however, are numerous and probably outweigh the advantages except in cases when the ordinary methods fail. The computations are more difficult and laborious, involving the use either of addition and subtraction logarithms, or of natural versines.

"Simultaneous" and "Double" Altitudes

The observed altitude of any body thus gives a line of position on which the observer is located. It cannot give his position on that line. But, if the altitude of a second body be taken, this

second observation will determine a second line of position, on which the observer must also be situated. Being thus on both lines at once, the observer must be at their point of intersection, and his absolute position is fully determined. The two heavenly bodies are used in a manner precisely similar to that in which two light-houses, or any two terrestrial objects, are used to obtain a ship's position by cross-bearings, and the method of "simultaneous" or "double" altitudes might aptly be called the method of "astronomical cross-bearings."

The only time that this simple method of cross-bearing can be used is at night, for in the day-time but one body, the sun, is visible. But at night many stars are visible, and, at almost any hour, two stars may be selected which will give admirable results. In selecting the stars, it is only necessary to pick out a pair, whose azimuths differ by more than 30°. For the lines of position will intersect at an angle equal to the difference of azimuths of the two bodies, and unless these lines "cut" at an angle greater than 30°, it will be difficult to determine the exact position of the point in which they cut. The nearer the "cut" is to 90° the more accurate the "fix."

This method of simultaneous altitudes is illustrated in the following figure. An observer in approximate latitude 39° observed two stars, one east, the other west, of his meridian. The first observation gave the Sumner line A C, the

second the line B C. As the observer must be on both lines, his true position was at C, their point of intersection. His assumed latitude, 39°, was in error by nearly 7′, and the first observation by

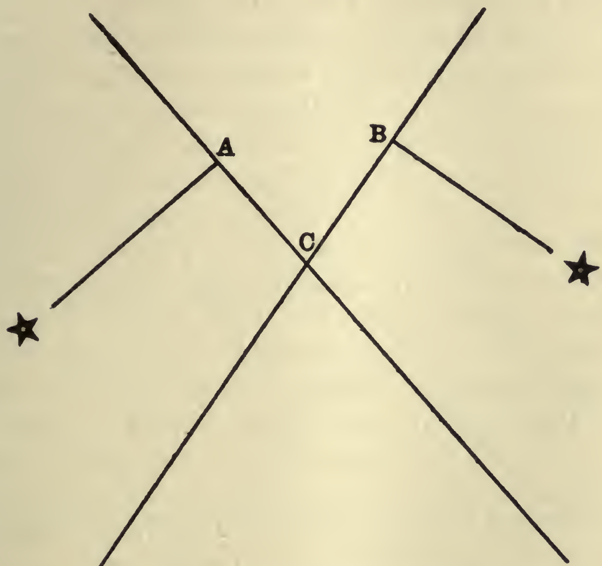

FIG. 14—Astronomical Cross-Bearings

itself would have located the ship at a point nearly 10′ from the true position.

The *Nautical Almanac* gives the positions of some 150 stars; so that at almost any hour of any night, a pair of these can be selected, which will serve for such "astronomical cross-bearings."

In the day-time, with only one body, the sun, visible, a modification of this method must be used. If the observer, or the ship, be stationary,

the modification is extremely simple. As the sun rises and sets it continually changes its azimuth, and, as the line of position is always at right angles to the direction of the sun, the line of position will rotate about the observer. If then an observation be made and a Sumner line drawn, and after an interval of an hour and a half or two hours a second observation be made, and a second Sumner line drawn, then these two lines will intersect at a considerable angle and an accurate "fix" will result. If, therefore, the ship be at rest for a period of two or three hours, the sun may be used at two different times in exactly the same manner as the two stars were used and the problem worked in a manner exactly like the former example. While the sun might be used in this way on land, at sea, however, such a case would be extremely rare, for vessels are seldom or never at rest; they are usually in rapid motion, and during the interval between observations a ship will have travelled many miles.

If the course of the ship and the distance travelled during the interval between the observations be known, then the first Sumner line may be carried forward parallel to itself and drawn through the position of the ship at the time of the second observation. This transference of the Sumner line might be better understood, if, according to Lecky, "we were to imagine it something the ship could carry with her, and drop *at the correct angle*, at the instant of making

the observation which was to cross it with the second Sumner line." The intersection of this

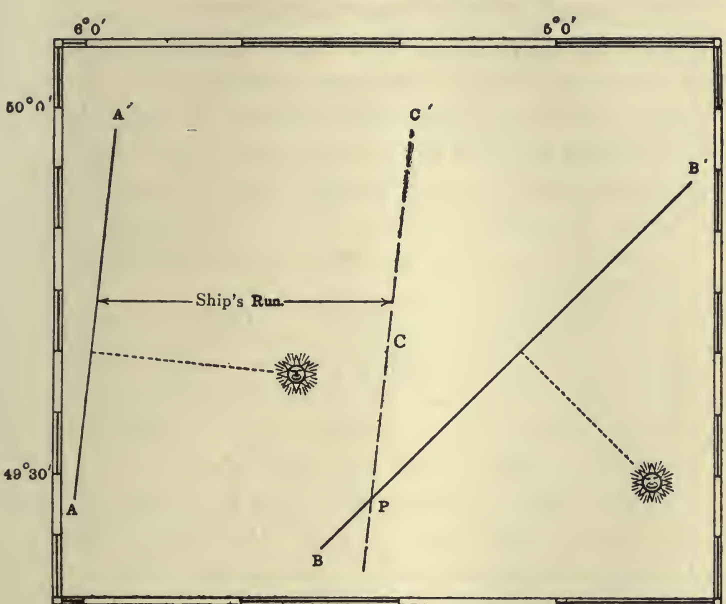

FIG. 15—Lines of Position

transferred line with the second Sumner line will be the true position of the ship at the time of the second observation. All of this is at once apparent by an inspection of Figure 15.

At about 8 A.M. a sight was obtained and the Sumner line A A′, drawn on the chart. This was worked with the approximate latitude by dead reckoning, *i.e.*, 49° 40′. At 10:30 A.M. a second observation was made. During the interval the ship had run twenty-five miles on a true east

course. The approximate position of the ship at the time of the second sight was, therefore, at C, and through C was drawn a line, C C′, parallel to the first Sumner line. Now it is evident that at 10:30 A.M. the ship must be somewhere on this line. The second sight was worked with the approximate latitude of C, *i.e.*, 49° 40′, and gave a Sumner line B B′ on which the ship must also be. Hence the true position of the vessel at the instant of the second observation was at P, the intersection of the two lines.

It is not always convenient to plot the Sumner lines on a chart. Again, the "fix" will not be good unless the chart be drawn to a large scale and the plotting very carefully done. Calculation is always better than curve plotting, and the latitude and longitude of the point of intersection of the two lines can be found just as quickly by calculations, and more accurately than by plotting on the chart. Yet the plotting of the curve on the chart should not be wholly given up in favor of calculation, for the actual seeing of the lines and their relative positions is often a great help; this is especially so when approaching land. Under these circumstances, as we have seen, a single Sumner line may give all the information desired, may show the distance of ship from shore, or may indicate the course to be sailed in order to make a good land-fall.

When the Sumner lines were first introduced into practical navigation, the methods of calcu-

lation were crude, and the actual working of the two sights of a "double altitude" was long and complicated. The necessary calculations frightened the navigator and it appeared to him as difficult and incomprehensible as the old method of "lunars." In the last few years, however, the methods have been much simplified, and tables have been computed by the aid of

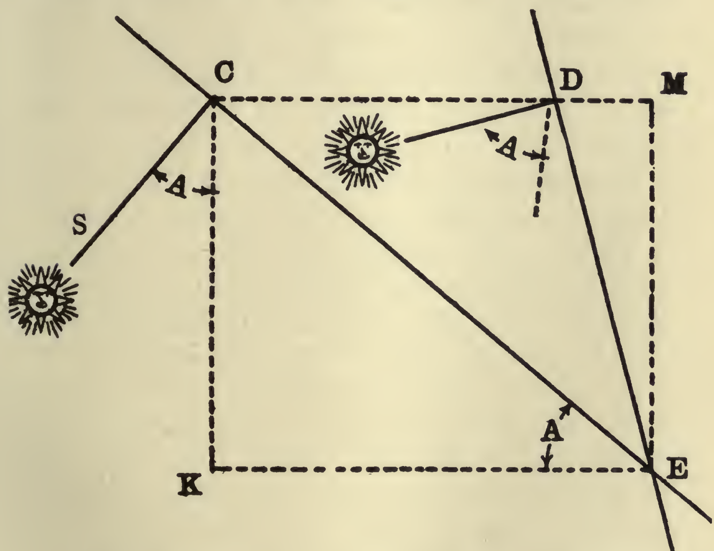

FIG. 16—Determining the Intersection of Sumner Lines

which the actual calculation is reduced to very few figures. To show the way in which these tables are formed and used, reference is had to the foregoing figure.

In this figure CE is the first Sumner line carried

forward for the run; DE is the second Sumner line, and E, their point of intersection, is the true position of the ship at the time of the second sight. C and D are the positions as determined with the latitude by dead reckoning from each of the two sights separately. From the figure it is at once seen that the angle CEK is equal to the angle SCK, for the sides of the first are respectively perpendicular to the sides of the second. This angle is equal to the true bearing or azimuth of the sun at the time of the first sight, and may be denoted by A. The similar angle for the second sight will be denoted by A'.

Now the ship ought to be at C, the latitude and longitude of which point are known by dead reckoning. By working out the second sight with this latitude by D.R., the ship was located at D, and CD, when reduced to the equator, will thus be the difference between the longitude by D.R. and the longitude as found from the second sight. The true position of the ship is at E, the intersection of the two Sumner lines, and CK is, therefore, the correction to the D.R. latitude, and EK is the departure correction to the D.R. longitude. Denoting the corrections to the latitude and longitude by ΔL and $\Delta\lambda$ respectively, we have from the figure, by simple trigonometry,

$$\Delta L = \frac{\lambda' - \lambda''}{\cot A \sec L - \cot A' \sec L}$$

$$\Delta\lambda = \Delta L \cot A \sec L$$

where λ' and λ'' are the longitudes as given by dead reckoning and by the second sight respectively.

Now to reduce this simple calculation to its lowest terms tables have been constructed which give the values of *cot A sec L* for various combinations of latitude and azimuth. Such a table is Table I, of Johnson's *On Finding Latitude and Longitude in Cloudy Weather* and also Table C of Lecky's *ABC Tables.* With the aid of such tables the computation is extremely simple and can be carried out in a few moments. Johnson expresses the above mathematical formulas in the form of three or four very simple rules. And when using these rules together with the tables, the navigator need never know that he is in reality making use of complicated formulas involving sines and cosines, tangents and cotangents.

NOTES AND PRACTICAL APPLICATIONS

1. *Formulas for Finding Sumner Lines:*

(*a*) Ordinary Method.

With the D.R. latitude and the observed altitude corrected for refraction, dip, and semidiameter, compute the hour angle by the formula:

$$\sin^2 \tfrac{1}{2}\, t = \frac{\cos S \sin (S-h)}{\cos L \sin P}$$

where

$$S = \tfrac{1}{2}\,(h + L + P)$$

Then:

1st *for the sun.*

Correct the observed chronometer time for chronometer error and for the equation of time, thus finding the Greenwich apparent time. The difference between this and the above found hour angle is the longitude of the required point on the circle of position, the latitude of which is equal to the latitude by D.R.

Take the azimuth of the sun from the azimuth tables and at right angles to the azimuth draw the Sumner line through the point as above found.

2nd *for a star or planet.*

If a *sidereal* chronometer be used, correct the observed time for chronometer error, thus finding the Greenwich sidereal time. To or from the right ascension of the body add (if west) or subtract (if east) the hour angle as above, and the result is the ship's sidereal time. The difference between these two times (Greenwich and ship's) is the required longitude. Proceed as in the case of the sun.

If a *mean time* chronometer be used, correct the observed time for chronometer error, thus finding the Greenwich mean time. Convert this into sidereal time by adding to it the sidereal acceleration (Table III, *N.A.*) and the sidereal time at Greenwich mean noon (taken from the *Nautical Almanac*); the result is the Greenwich sidereal time.

To or from the right ascension of the body add (if west) or subtract (if east) the hour angle as above, and the result is the ship's sidereal time. The difference between these two times (Greenwich and ship's) is the required longitude.

Proceed then as in the case of the sun.

(*b*) Method of Marcq St. Hilaire, "New Navigation."

1st *for the sun.*

Correct the observed chronometer time for chronometer error and equation of time, thus finding the Greenwich apparent time. Subtract from this the D.R. longitude and thus get the ship's apparent time, or the sun's hour angle, *t*. With this and the D.R. latitude

compute a zenith distance by the formula:

$$\text{hav } Z' = \text{hav } (L - D) + \cos L \cos D \text{ hav } t$$

using tables of natural haversines.

Correct the observed zenith distance for dip, refraction and semi-diameter, and find the difference between it and the computed zenith distance. Find the azimuth of the sun from the Azimuth Tables. On the chart plot the position by D.R. and from this point draw a line in the direction of the azimuth. On this line lay off from the D.R. position a distance equal to the difference of zenith distances, in the direction of the sun, if the computed zenith distance be greater than the observed.

Through the point thus found draw the line of position at right angles to the azimuth.

2nd *for a star or planet.*

Correct the observed chronometer time for chronometer error thus finding the Greenwich mean time. Convert this into sidereal time by adding to it the sidereal acceleration (Table III, *N.A.*) and the sidereal time at Greenwich mean noon (taken from the *Nautical Almanac*). The result is the Greenwich sidereal time.

From this subtract the D.R. longitude to find the ship's sidereal time. From the ship's sidereal time, subtract the right ascension of the body and the result is the hour angle, t.

Then proceed as in the case of the sun.

2. *Formulas for "Simultaneous" or "Double" Altitudes:*

(*a*) By Plotting on Chart.

If two stars have been observed at the same time find from each observation the corresponding Sumner lines. Plot them on the chart, and the point of intersection is the true place of the ship.

If two observations of the sun have been made find from each observation the corresponding Sumner lines. Plot these on the chart, then move the first line forward parallel to itself by an amount equal to the run of the ship during the interval between the observations; the intersection of this line with the second Sumner line is the true place of the ship at the moment of the second observation.

(*b*) By Calculation.

Enter Johnson Table I, or Lecky's Table C, with the azimuths and latitudes of the first and second observations and take out the quantities, c and c'.

Then the corrections to the D.R. latitude and longitude will be given by,

$$\Delta L = \frac{\lambda' - \lambda''}{c - c'}$$

$$\Delta\lambda = \Delta L \times c$$

where λ' and λ'' are the longitudes as given by D.R. and by the second sight respectively.

If these tables are not at hand, then c and c' can be computed from the formulas

$$c = \cot A \sec L$$

$$c' = \cot A' \sec L'$$

3. *Practical Examples:*

(*a*) Sumner Line—Observation of the Sun.

At sea,[1] Nov. 9th, in D.R. latitude 34° 20′ N. the altitude of the sun was 17° 44′ 10″ at Greenwich mean time, $8^h\ 15^m\ 1^s$. Required the line of position.

Obsd. alt.	17°	44′	10″
Index	+	2	40
Dip	−	4	3
Table IV	+	13	8
Table V	+		11
h	17°	56′	6″
L	34	20	
P	106	56	5
	2)159°	12′	11″
S	79	36	5
S− *h*	61	39	59

cos S	9.25645
sin $(S-h)$	9.94458
sec L	0.08314
cosec P	0.01925
haversine	9.30342

Apparent time	3^h	33^m	9^s
Eq. of time	−	16	4
Mean time	3	17	5
Gr. m. t.	8	15	1
Longitude	4^h	57^m	56^s
	74°	28¾′	

Declination

At Gr. m. n.		16°	50′	10″	S.
Corr. for $8^h.25$	—		5	55	
D		16°	56′	5″	S.

Equation of Time

At Gr. m. noon	16^m	$5^s.7$
Corr. for $8^h.25$	—	1 .7
Eq. (add to mean)	16^m	4^s

From Davis's Tables,
⊙'s azimuth = N. 128° W.

[1] This and the following problem were adapted from Sturdy's *Practical Aid to the Navigator*, the dates being changed to 1908.

Hence the line of position passes through the point whose latitude is 34° 20′ N. and longitude 74° 28¾′, and the line runs N. 38° W., or at right angles to the true bearing of the sun.

The plotted line of position passes a trifle to the north of Cape Hatteras, the bearing of which is thus very closely determined.

(*b*) Sumner Line—Observation of a Star.

At sea, Nov. 9th, after running 32 miles north, in D.R. latitude 35°, the altitude of Aldebaran was 40° 14′ 30″ at Greenwich mean time $14^h\ 47^m\ 13^s$. Required the line of position.

Obsd. alt.	40°	14′	30″	*Aldebaran*			
Index	+	2	4	Rt. ascension	4^h	30^m	38^s
Dip	−	4	3	Declination	16°	19′	30″ N.
Table IV	−	1	10				
h	40°	11′	21″				
L	35	0		Gr. mean time	14^h	47^m	13^s
P	73	40	30	Reduction	+	2	26
	2)148°	51′	51″	Sid. interval	14	49	39
S	74	25	56	Sid. t. at G.m.n.	15	13	1
S–h	34	14	35				
cos *S*	9.42877			Gr. sid. time	30^h	2^m	40^s
sin (*S–h*)	9.75028						
sec *L*	0.08664			From Davis's Table,			
cosec *P*	0.01787			*'s azimuth = N. 98½° E.			
haversine	9.28356						

Hour angle	3^h	27^m	58^s
Rt. ascension	4	30	38
Local sidereal time	1^h	2^m	40^s
Gr. sidereal time	6	2	40
Longitude	5^h	0^m	0^s

Hence the line of position passes through the point whose latitude is 35° N. and longitude 75° W. and the line runs N. 8½° E., at right angles to the true bearing of the star. As this line runs so nearly north and south it fixes the longitude and the distance off shore with great accuracy.

(*c*) New Navigation.[1]

On Sept. 28th, in D.R. latitude 46° 20′ N. and longitude 29° 57′ W., at G. m. t., $13^h\ 39^m\ 1^s$, the observed altitude of α Ursæ Majoris, "Dubhe" was 19° 26′ 46″; height of eye 20 feet. Find the Sumner line.

*'s R.A.	10^h	58^m	4^s	cos D	9.66807
Decl.	+62°	14′	52″	cos L	9.83914
				hav t	9.98960
G. m. t.	13^h	39^m	1^s		
Sidereal acc.	+	2	14		9.49681
Sid. t. at G. m. n.	12	27	26		+0.31391
				nat. hav(L–D)	+0.01916
G. sid. time	2^h	8^m	41^s		
Longitude	1	59	48	nat. hav Z'	+0.33307
				Z'	70° 29′ 50″
Local sid. time	0^h	8^m	53^s		
*'s R. A.	10	58	4	*For Azimuth*	
Hour Angle	10^h	49^m	11^s	Lecky's A	−3.284
				" B′	−6.332
Obsd. alt.	19°	26′	46″		
Dip	−	4	24		−9.616
Table IV	−	2	44		
				Table C. Az. = N. 8½° E	
True alt.	19°	19′	38″		
Z	70	40	22		
Z'	70	29	50		
$Z - Z'$	+	10′	32″		

[1] Adapted from Norie.

Thus through the position by D.R. draw a line running N. 8½° E.; on this lay off a distance equal to 10½′ towards the *south* for the computed zenith distance is less than the observed. Through the point thus found draw the Sumner line running N. 98½° E. The ship will be somewhere on this line.

This observation could not be worked in the usual way, for the azimuth of the star is so small that an error of 1′ in the assumed latitude would introduce an error of 9½′ in the resulting longitude. As the latitude was over 10′ in error, the longitude as found by the usual formula would have been over 1° 50′ in error. If one is *obliged* to use such an observation, then the method of "New Navigation" should be used.

(*d*) Astronomical Cross Bearings—Observations of the Sun.

At sea, June 27th,[1] in D.R. latitude 49° 40′ N., the following altitudes of the sun were taken to determine the ship's place by the intersection of lines of position. In the interval between the observations the ship ran twenty-five miles on a true east course.

	First Sight	*Second Sight*
Greenwich m. t.	$8^h\ 24^m\ 14^s$ A.M.	$10^h\ 51^m\ 40^s$ A.M.
True altitude	36° 24′ 20″	58° 9′ 20″
Latitude by account	49° 40′ N.	49° 40′ N.
Sun's declination	23° 19′ 37″ N.	23° 19′ 24″ N.
Equation of time	+ $2^m\ 48^s$	+ $2^m\ 49^s$
Azimuth	N. 95½° E.	N. 135° E.

[1] Adapted from Bowditch—A comparison with the original will show the reduction in calculation due to the use of modern methods.

h	$36°\ 24'\ 20''$	$58°\ 9'\ 20''$
L	$49\ 40$	$49\ 40$
P	$66\ 40\ 23$	$66\ 40\ 36$
	$2)152°\ 44'\ 43''$	$174°\ 29'\ 56''$
S	$76\ 22\ 22$	$87\ 14\ 58$
$S-h$	$39\ 58\ 2$	$29\ 5\ 38$
cos S	9.37219	8.68106
sin $(S-h)$	9.80778	9.68686
sec L	0.18894	0.18894
cosec P	0.03703	0.03702
haversine	9.40594	8.59388
Apparent time	$7^h\ 57^m\ 33^s$ A.M.	$10^h\ 28^m\ 35^s$ A.M.
Equation of time	$+\ 2\ 33$	$+\ 2\ 49$
Local m. t.	$8^h\ 0^m\ 21^s$	$10^h\ 31^m\ 24^s$
Greenwich m. t.	$8\ 24\ 14$	$10\ 51\ 40$
Longitude	$+\ 23^m\ 53^s$	$+\ 20^m\ 16^s$
	$5°\ 58\frac{1}{4}'$ W.	$5°\ 4'$ W.

These two lines of position are plotted on the chart (Fig. 15) as A A′ and B B′ respectively. During the interval between the observations the ship ran twenty-five miles due east, and, therefore, the first Sumner line is shifted parallel to itself and carried to C C′, twenty-five miles east of the first position. This transported line and the second Sumner line intersect at P, in latitude 49° 28′ N. and longitude 5° 23′ W., which is the true position of the ship at the time of the second observation.

If it be inconvenient to plot the lines on a chart, the position of the ship can be found by calculation as follows: the quantities c and c′ being taken from Lecky's Table C.

			From Lecky's Table C
1st longitude	+	5° 58¼′	$c = 0.15$
Ship's run		38½′	$c' = 1.54$
λ'	+	5° 19¾′	$c' - c = 1.39$
λ''	+	5 4	

$$1.39)\ 15'.75\ (11'.33 = \Delta L$$

$$11'.33 \times 0.15 = 1.'7 = \Delta\lambda'$$
$$11'.33 \times 1.54 = 17'.4 = \Delta\lambda''$$

2d longitude	+	5° 4′.0
Correction		17′.4
True longitude	+	5° 21′.4
D. R. latitude	+	49° 40′
Correction	−	11 ⅓
True latitude	+	49° 28⅔′

And these results by calculation agree within a fraction of a mile with position as found by chart plotting.

CHAPTER VIII

LATITUDE

THE method of finding one's position at sea by means of lines of position is fundamental; other methods are but special cases of this most general and powerful problem. It utilises to the fullest extent observations taken at almost any hour of the day, and to that method the would-be navigator should devote his earnest attention. Lord Kelvin, in a lecture at Glasgow said, "that it would be the greatest blessing to navigators, both young and old, if every other method of ordinary navigation could be swept away."

In this statement there is but little exaggeration. With a thorough knowledge of Sumner lines and their use, an officer may safely navigate a ship around the world; he need never use any other method. Of no other method can this be said. Yet, on the other hand, in special cases and situations, other and older methods may furnish all the data required, and at the same time be more readily and quickly applied. Yet it must always be borne in mind that these older methods are but special cases of the general method of

lines of position. Such special cases are those in which the latitude alone is required—the time honoured "noon sight" being the most important.

The line of position, the Sumner line, is always at right angles to the sun's (or star's) true bearing. Now at noon the sun bears south, is on our meridian, and consequently the Sumner line runs due east and west. An observation made at this time fixes accurately the ship's position on a circle of latitude, shows its exact distance north or south of the equator, but fails utterly to determine anything in regard to its longitude. A glance at the original figure in which a circle of position was shown, brings out clearly that, when the sun is on the meridian, the observer will be in the same longitude as the subsolar point, and that the observer's latitude will differ from the latitude of the subsolar point by the measured zenith distance. The latitude of the observer is the latitude of the subsolar point plus the observed zenith distance. But the latitude of the subsolar point is equal to the declination of the sun, hence the simple rule,

"The latitude of the observer is equal to the declination of the sun or star when on the meridian plus the observed zenith distance."

Putting this in the form of an equation it may be written,

$$L = D + Z$$

where the letters have their usual significance.

This equation and its equivalent statement in words are perfectly general and may be applied to either north or south latitude, provided that north latitude is always given the positive (plus) sign and south latitude the negative (minus) sign. Similar conventions are made in regard to the declination and zenith distances: declinations are positive when north, negative when south; zenith distances are positive when the sun or star is south of the zenith, negative when the sun or star is north of the zenith. With these simple conventions, the above rule or formula can be used in every possible case. Many text-books and lecturers try to distinguish and separate the different cases arising when the observer is north or south of the equator; giving complicated diagrams and instructions, showing when to add and when to subtract the zenith distance.

All this is confusing, the one simple rule applies to every case, provided the conventions are recognized. To repeat:

Latitude and Declination	positive (plus) when north of equator; negative (minus) when south of equator.
Zenith distance	positive (plus) when the sun or star is south of the zenith; negative (minus) when the sun or star is north of the zenith.

The simple and direct application of this method is to measure with any instrument the zenith distance of any known heavenly body when on the meridian. Then a simple application of the above rule gives at once the latitude of the observer. It is the simplest and most accurate method of finding the latitude; it is used with the fixed instruments of an observatory, or with the portable instruments of travellers by sea and by land. The method and the formula are identical, whether the sun or a star be used, identical in principle and in detail. In the illustration the words sun and subsolar point have been used, but the sentences and the rules would be equally true if for these were substituted, "star" and "substellar" point. Some text-books attempt to distinguish cases in which stars and planets are observed from that in which the sun is used. This is certainly confusing: there is but one case and one rule—and this one rule applies to every possible observation of sun, moon, planet, or star.

In a permanent observatory and with fixed instruments the measurement of the meridian zenith distance is extremely simple. In the meridian circle, for example, a spider line marks the position of the meridian, and when the star crosses this thread or line, the observer knows that it is on the meridian, and he then, at that instant, measures its zenith distance. But at sea, or on land, when the observer has but a sextant, the meridian is not marked and it is difficult to

tell the exact instant at which the zenith distance should be measured. In such cases it is customary to assume that the body is on the meridian, when it reaches its greatest altitude. The star or sun is watched as it approaches the meridian, and its slowly increasing altitude noted. As the body reaches the meridian the altitude remains stationary for a moment, then begins to decrease—and this moment of stationary altitude is the moment of meridian passage. Now while this is absolutely true for an observer in a fixed position, and when observing a star, it is not strictly correct for an observer on a rapidly moving ship, nor for an observation of the sun or moon.

The sun's declination is constantly changing, hence the greatest altitude will always be reached a little before or a little after meridian passage. Between December and June the declination of the sun is increasing, and to an observer in north latitude it will, during these months, reach its greatest altitude a few seconds after it crosses the meridian. Just at the meridian the sun is moving almost horizontally, and at this part of its daily path, the increase in the declination is sufficient to overcome the decrease in altitude due to its setting. During the period from June to December the declination is decreasing, and hence such an observer would find the greatest altitude a few seconds before meridian passage. This difference between the sun's greatest and meridian altitude,

is, however, very slight and may be, and is, neglected in all navigational problems.

The motion of the ship, however, may introduce a complication which must be taken into account. If the vessel be on a north or south course, and be in rapid motion, then the apparent rise or fall of the sun or star, due to the observer's motion, may counterbalance the real rise or fall, and the maximum altitude may differ considerably from the meridian altitude. A twenty knot steamer, running due south, would, by its motion, cause the sun to apparently rise $1'$ of arc for every 3^m of time; and it often requires five or six minutes for the sun to change its altitude by this amount. The steamer's motion would cause the sun to apparently rise for some minutes after it had actually passed the meridian, and the measured altitude might well differ $2'$ or $3'$ from the true meridian altitude. In such extreme cases, it is best to calculate beforehand the chronometer time at which the sun or star should cross the meridian, and to measure the altitude at this moment, and to use the altitude thus found in place of the usual maximum altitude.

There is only one apparent exception to the simple rule, given above, for finding the latitude from the meridian zenith distance, and this exception is apparent not real. It is, when the altitude of a circumpolar star has been taken at its lower culmination. In middle and high latitudes there are a number of bright stars which never set.

These remain above the horizon at all times, travelling about the pole in small circles, and they may, therefore, be seen to cross the observer's meridian twice each day—once above the pole and once below the pole. These two meridian passages are of course twelve hours apart and it is seldom that the two passages can actually be observed on the same day. Now it is evident that the altitude, or the zenith distance, of the star will be radically different at these two passages of the meridian, and, consequently, the formula for the latitude cannot be used in the same identical form in the two cases. The latitude of a place is its distance from the equator, the distance from that point of the equator, which is on the meridian, to the zenith; the declination of a star is also its distance from the equator. When the star crosses the meridian above the pole, these two points, that from which latitude is measured and that from which declination is measured, coincide; but when the star crosses the meridian below the pole, these two points are 180° apart. When we use any formula involving measurements, we must always start the measurements from the same point, or allow for the difference. In the case of the star at lower culmination, if the declination were measured from the same point as the latitude, then such declination would be equal to 180° minus the tabulated declination of the star. If such corrected declination be used then the original formula, or rule, is

correct and may be used without change of any kind.

We have, therefore, the simple rule for observing stars at their lower culmination:—apply the simple formula or rule, using the supplement of the declination ($180° - D$) instead of the tabular declination.

Attention should be called to the fact that when such observations are made, the *smallest*, instead of the *greatest* altitude is used. The star swings around its circle, coming to the meridian from the west, and reaches the lowest part of its course at the moment it crosses the meridian.

Ex-Meridian Altitudes

Clouds often cover the sky and prevent the navigator obtaining a meridian altitude of the sun or of a star. But perhaps half an hour or so, before or after the meridian passage, the sky may clear up and allow a good sight. An observation made so near the meridian is of no value for longitude, but is for latitude, and it becomes of importance to be able to use such an observation in a simple and ready manner. The method commonly used for this purpose is variously known, "ex-meridian," "reduction to the meridian," etc. It is applicable whenever the altitude of the sun or the star has been measured, when the body is not over three quarters of an hour from meridian passage, and when the local time is known with considerable accuracy. In this latter regard

this ex-meridian method does not compare favourably with the simple way of determining the latitude from the meridian altitude, which is independent of the time; independent of any possible lurking error in the chronometer.

The ex-meridian method is simple in principle and easily understood. It consists, merely, in calculating from the observed altitude of the body and from the local time at which the observation was made the altitude the body should have when it reaches the meridian. For example, suppose that when a body is near the meridian, its altitude increases $\frac{1}{4}'$ during each five (5) minutes of time, and further, suppose that its altitude was measured twenty (20) minutes before the time at which it should cross the meridian, then a simple multiplication shows, at once, that the meridian altitude will be $1'$ greater than the measured altitude. The meridian altitude may thus be calculated from the observed altitude, by adding to it a certain quantity called the "reduction to the meridian." This reduction depends primarily upon the distance from the meridian at which the observation is made, and if the rate at which the altitude increases be known, then the reduction can be tabulated for different numbers of minutes between the observation and meridian passage.

Of course such a simple case as the above can seldom happen, for the altitude does not in general increase uniformly with the time. For an

observer on the equator it might happen twice during the year, in March and September, when the sun's declination is zero. On these days an observer so situated would see the sun rise vertically in the east, pass through the zenith, and set vertically in the west. The altitude would change uniformly 15° each hour, the sun remaining exactly twelve hours above the horizon. But for an observer at any other point of the earth's surface, or for any observer on any other day of the year, the increase or decrease in the altitude of the sun will not be uniform. The altitude changes most rapidly in the early morning and late afternoon hours, while near noon the change in altitude gradually becomes less and less. At noon, as the sun crosses the meridian it travels for a moment parallel to the horizon, and there is, for a few moments, no appreciable change in the altitude.

Still further, the rate, at which the altitude increases when the sun is near the meridian, varies with the position of the observer on the earth, and with the position of the sun in the heavens. It depends upon the latitude of the observer and upon the declination of the sun. Elaborate formulas have been derived for computing this rate of change in altitude and the corresponding reduction to the meridian, for an observer in any latitude and for any heavenly body. These formulas are all too complicated for ordinary use in navigation, but from them

tables have been computed, which are very simple and of great value to the navigator. The latest of these tables are those of Lieut.-Commander Rust, U. S. N.

Practically all these various tables depend upon the formula for the reduction as first given by Delambre. By this formula the reduction to the meridian in seconds of arc is equal to:

$$+\frac{\cos L \cos D}{\sin (L-D)} \times \frac{2 \sin^2 \frac{1}{2} t}{\sin 1''}$$
$$-\left(\frac{\cos L \cos D}{\sin (L-D)}\right)^2 \times \frac{2 \sin^4 \frac{1}{2} t \cot (L-D)}{\sin 1''}$$

where t is the time from meridian passage expressed in seconds of time. The two terms of this formula are called the "1st Reduction" and the "2d Reduction," and are denoted by R_1 and R_2 respectively. The second term is small in comparison to the first and generally may be neglected, unless extreme accuracy is desired. The whole reduction may be put into the simple form,

$$R = R_1 - R_2$$

and when the time is counted in minutes from meridian passage and the reduction is given in seconds of arc, the two may be put into the form:

$$R_1 = A\ t^2$$
$$R_2 = \sin^{-1} B$$

in which,

$$A = \frac{1''.9635 \cos L \cos D}{\sin\ (L-D)}$$
$$B = \tfrac{1}{2} \sin^2 A \cot\ (L-D)$$

These values show that, near the meridian, the altitude of a body varies with the square of time. In two minutes the altitude changes four times, and in three minutes nine times as much as in one minute. *A* is the change in the altitude in one minute, in the one minute either before or after meridian passage. At a given locality on the earth its value depends solely upon the declination of the observed body, and its various values can be computed once for all with different combinations of latitude and declination. For example in New York City, the latitude of which is approximately 41° N., the altitude of the sun, when on the equator, will change at the rate of 2″.26 in the minute previous to crossing the meridian. In the latter part of June when the sun is in 23° north declination, this rate of change is nearly doubled, becoming 4″.41. This 4″.41 is the reduction for one minute; for two minutes it will be four times and for four minutes sixteen times this, or 70″.

With the values of *A* corresponding to different combinations of latitude and declination, tables can thus be arranged which shall give directly the reduction corresponding to any hour angle. Such a table is that of Rust's, which gives the reduction at intervals of four (4) minutes. In ordinary cases the value of the reduction can be taken directly from this table by simple interpolation; that is, when the hour angle is 4½, 5, or 7 minutes, or any value not directly given, the corresponding value of the reduction can be found to the nearest

minute of arc from the table. But if extreme accuracy is required, then, for any such odd hour angle, the value of the reduction should be calculated from the tabular value of the A, or Δh as Rust calls it, by multiplying it by the square of the number of minutes in the hour angle.

The second term of the reduction is very small as compared with the first, generally amounting to only a few seconds. At the foot of each column of his tables Rust gives the hour angle in minutes of time, at which the second term amounts to about three quarters ($\frac{3}{4}'$) of a minute of arc. If the hour angle is less than this the second term can be omitted, if greater it must be taken into account. This second term could be computed and tabulated in a manner exactly similar to the first term, but, on account of its small size Rust found it more convenient to give its values in the form of diagrams. The curves and diagrams are so simple that they require no explanation. In low latitudes this second term of the reduction becomes relatively large at small hour angles, while in high latitudes altitudes of the sun taken nearly an hour from the meridian may be reduced with considerable accuracy without taking the second term into account.

In using the tables we must know the approximate latitude of the ship and the hour angle of the sun or star at the moment of observation. The approximate latitude is found by dead reckoning, the nearest degree being sufficient in most

cases. The hour angle, on the other hand, must be determined with accuracy, and is found by noting the chronometer time at which the observation was made. When the sun is observed the chronometer gives directly the Greenwich mean time, from which, by adding the approximate longitude and the equation of time, the local or ship's time may be found, and this ship's time is the hour angle of the sun at the moment of observation. A pretty accurate knowledge of the longitude is, therefore, necessary in applying this ex-meridian method.

When the meridian altitude has been found by this method, then the process of finding the latitude therefrom is identical with the ordinary method of finding the latitude from a noon sight. The computed meridian altitude is treated exactly as though it had actually been observed at noon, or when the body was on the meridian. But it must always be remembered that the resulting latitude is the latitude of the ship at the moment the observation was made, and is not the latitude at noon. If the latitude at noon be required, while an ex-meridian observation only is available, then the latitude as found from the ex-meridian method must be corrected for the run of the ship between the moment of observation and noon. Knowing the course made and the rate at which the ship is moving through the water, this correction can readily be found from the traverse tables.

The value of the reduction depends upon the hour angle, and our knowledge of this depends in turn upon the chronometer and upon the assumed longitude, or longitude by dead reckoning. If, therefore the longitude be somewhat doubtful, it might be useful to know what dependence can be placed upon the latitude found by an ex-meridian observation. The error in the reduction, or the resulting latitude due to an assumed error of (1^m) one minute in the hour angle, can readily be taken from the tables. The tables give the reduction for intervals of four minutes, and by taking out the values of the reduction for the assumed hour angle and also for an hour angle four (4^m) minutes greater, the difference between these two values will be the error in the latitude for an assumed error of four (4^m) minutes in hour angle.

For some reasons ex-meridian observations are even preferable to meridian observations. The observation itself takes less time, there is no waiting for the sun to reach its maximum altitude—an altitude is measured and the corresponding time noted and that is all there is to the observation. As this is instantaneous, the motion of the ship has no effect, such as it has in the case of the meridian altitude.

Latitude by an ex-Meridian Altitude of the Pole Star

If the Pole Star were situated exactly at the north pole, the problem of finding the latitude

would be very simple. For the latitude of any place is equal to the altitude of the north pole above the horizon, and, if the Pole Star were at the pole, then this altitude could always be directly measured. But, as a matter of fact, Polaris is a little more than a degree ($1° 11'.4$) from the pole, and it, therefore, describes each day a small circle about the pole. As it travels around and around in this circle, it will cross the meridian twice, once above and once below the true pole, and twice each day it will have the same altitude as the true pole. At these latter times the altitude of Polaris is directly equal to the latitude of the place. At all other times its altitude will differ from the latitude, being either a little greater or a little less.

At any time, therefore, the measured altitude of Polaris will differ from the latitude by an amount which depends solely upon the size of this circle, which the star describes about the pole, and the position of the star in this circle. Thus, if the radius of the circle and the hour angle of Polaris be known, this difference between the altitude and the latitude can be computed. The radius of the circle varies a little from year to year, owing to precession, but its value at any date is given in the *Nautical Almanac*. The hour angle of Polaris can easily be found from the time at which the observation is made and the known position of the star in the heavens. For a single day the radius of the circle is sensibly constant, but the

hour angle varies from minute to minute, changing by 360° in each twenty-four sidereal hours.

The correction, therefore, depends primarily upon the hour angle, and its value for different

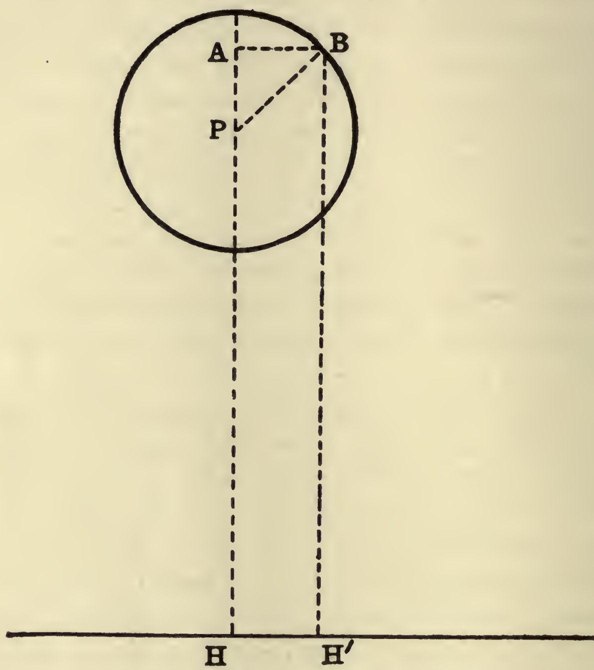

FIG. 17—Latitude by Pole-star Observations

hour angles can be tabulated once for all. Such a table is given in the *Nautical Almanac* each year, and the use of this table reduces the work of finding the latitude to a minimum. The navigator has merely to measure the altitude of Polaris and note the time on his chronometer. This chronom-

eter Greenwich mean time is then transformed into the corresponding local sidereal time, the difference between which and the star's right ascension is the hour angle required. With this hour angle the table gives at once the correction to be applied to the observed altitude.

This Pole-Star method is nothing more nor less than a special case of ex-meridian altitudes. The observed altitude of this star could be reduced by the ordinary formula and methods by which ex-meridian observations are reduced. Owing, however, to its high declination the ordinary tables do not extend so far, and the special table is, therefore, computed and inserted in the almanac.

NOTES AND PRACTICAL APPLICATIONS

1. *Formulas for Latitude.*

(*a*) Meridian Altitudes.

Correct the meridian altitude of any body, sun, star, or planet for refraction, dip, and semi-diameter, then the latitude can be found by,

$$L = D + Z$$

where,

D and L are positive (+), when north of equator; negative (−), when south of equator.

Z is positive (+), when the sun (or star) is south of the zenith; negative (—), when the sun (or star) is north of the zenith.

When a circumpolar star is observed at lower culmination, then use the supplement of the declination ($180° - D$) in place of D.

(*b*) Ex-Meridian Altitudes.

Measure the altitude of any body, sun, star, or planet, when near, but not on, the meridian, and note the chronometer time of observation. From the chronometer time and the D.R. longitude find the hour angle of the body at moment of observation.

With this hour angle, the D.R. latitude and the declination enter any ex-meridian table and take out the reduction. Correct the observed altitude for refraction, dip, and semi-diameter, and add to the result the reduction. This will give the meridian altitude.

With this reduced meridian altitude find the latitude, at the moment of observation, by means of the formula in Section (*a*), above.

(*c*) Altitude of the Pole Star.

Measure the altitude of the Pole Star and note the chronometer time of observation. Correct this time for chronometer error and transform into sidereal time by adding the sidereal acceleration (Table III, *N. A.*) and the sidereal time of Greenwich mean noon (from the *Nautical Almanac*). Subtract from this the D.R. longitude and thus find the ship's sidereal time. The difference between this and the star's right ascension is the hour angle.

With this hour angle enter the table in the *Nautical Almanac* and find the "correction." Correct the observed altitude for refraction, and dip, and apply to the result the tabular "correction," the result is the latitude.

2. *Practical Examples.*

(*a*) Latitude by Meridian Altitudes.

At sea, Nov. 26th, in D.R. latitude 40° 20′ N. and D.R. longitude 72° 30′ W., the observed meridian altitude of the sun's lower limb was 28° 32′; height of eye, 30 feet; index correction + 1′ 30″. Required the latitude.

Obsd. alt.	28° 32′ 0″		☉'s *Declination.*	
Index	+ 1 30		At Gr. app. n.	20° 55′ 47″S.
Dip	− 5 23		Corr. for $4^h.83$	− 2 18″
Table IV	+ 14 21		*D*	20° 58 5″S.
Table V	+ 11			
h	28° 42′ 39″			
Z	+61° 17′ 21″			
D	−20 58 5			
L	+40° 19′ 16″			

The true latitude is, therefore, 40° 19¼′ N.

At sea Aug. 22d, in D.R. latitude 43° 0′ N. and D.R. longitude 67° 30′ W., the observed meridian altitude of the sun's lower limb was 58° 20′ 20″; height of eye, 8 feet; index correction, − 1′ 10″. Required the latitude.

Obsd. alt.	58° 20′ 20″		☉'s *Declination.*	
Index	− 1 10		At Gr. app. n.	11° 50′ 31″N.
Dip	− 2 47		Corr. for $4^h.5$	− 3 47
Table IV	+ 15 30		*D*	11° 46′ 44″N.
Table V	− 11			
h	58° 31′ 42″			
Z	+31° 28′ 18″			
D	+11 46 44			
L	+43° 15′ 12″			

The true latitude is, therefore, 43° 15¼′ N., or the ship is fifteen miles to the north of its position by account.

(*b*) Latitude by Ex-Meridians.

At sea, Nov. 26th, in D. R. latitude 40° 18′ N. and D.R. longitude 72° 28′ W., the observed altitude of the sun's lower limb was 28° 17′ 30″

at 4^h 12^m 25^s Gr. m. t.; index correction, $+1' 30''$; height of eye, 30 feet. Required the latitude.

Obsd. alt.	$28° 17' 30''$
Index	$+ \quad 1 \quad 30$
Dip	$- \quad 5 \quad 23$
Tables IV & V	$+ \quad 14 \quad 30$
True alt.	$28° 28' 7''$
Reduction for $24^m.9$	$+ \quad 16 \quad 40$
Meridian alt.	$28° 44' 47''$
Z	$+61° 15' 13''$
D	$-20 \quad 58 \quad 5$
L	$+40° 17' 8''$

⊙'s *Declination.*

At Gr. app. n.	$20° 55' 47''$ S.
Corr. for 4^h. 83	$- \quad 2 \quad 18$
D	$20° 58' 5''$ S.

Eq. of Time.

At Gr. m. n.	12^m $38^s.6$
Corr. for $4^h.2$	$-$ 3.3
Add to mean	12^m 35^s.

Hour Angle.

Greenwich m. t.	4^h 12^m 25^s
Longitude	4 49 52
Local m. t.	23^h 22^m 33^s
Eq. of time	+ 12 35
Apparent time	23^h 35^m 8^s
⊙'s hour angle	24^m 52^sE.

The latitude at the time the observation was made was, therefore, $40° 17'$ N. The ship's course was N. $45°$ W. (true) and her speed 12 knots. In 25^m, therefore, she sailed 5 miles, changing her latitude by $2'.5$.

Hence this observation gave for the noon latitude $40° 19\frac{1}{2}'$ N., which agrees closely ($22''$) with that found at noon.

It is often advantageous to make a series of measures at intervals of five or ten minutes for half an hour or so before or after noon, and reduce each measure as a separate ex-meridian sight, and then take the mean of

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all as the definitive latitude. The sextant should be set at a whole number of degrees and minutes, and the moment the sun reaches this altitude should be noted on the chronometer. The sextant should then be set at from 10′ to 15′ greater altitude, and the time again noted. This process should be repeated with a diminishing interval until the sun has passed the meridian, when the process should be reversed.

At sea Aug. 22d, in D.R. latitude 43° 0′ N. and D.R. longitude 67° 30′ W., the following altitudes of the sun's lower limb were observed, and the corresponding times noted: index correction, − 1′ 10″; height of eye, 8 feet. Course E. ½ S., speed 5 knots.

Obsd. Altitudes	*Chronometer Times*	*Time from Noon*
57° 45′	5^h 04^m 50^s	-27^m 56^s
58 0	11 30	−21 16
58 15	22 4	−10 42
58 20 20″	33 18	+ 0 32
58 15	43 46	+10 0

		☉'s Declination.	
Index corr.	− 1′ 10″	At G. app. noon	11° 50′ 31″ N.
Dip	− 2 47	Corr. for $4^h.5$	− 3 47
Tables IV & V	+15 19		
		D	11° 46′ 44″ N.
Correction	+11′ 22″		

Equation of Time.

At G. app. noon	2^m 49^s
Corr. for $4^h.5$	− 3
Equ. (add to app.)	2^m 46^s

Local app. noon.	0^h	0^m	0^s
Equation	+	2	46
M. t. of app. noon	0^h	2^m	46^s
Longitude	4	30	
G. m. t. of app. noon	4^h	32^m	46^s

Time from Noon	*Corrected Altitude*	*Reduction "Rust"*	*Meridian Altitude*
-27^m 56^s	57° 56′ 22″	35′ 20″	58° 31′ 42″
−21 16	58 11 22	20 46	32 6
−10 42	58 26 22	5 13	31 35
+ 0 32	58 31 42		31 42
+10 0	58 26 22	4 45	31 7
		Average	58° 31′ 38″
		Z	+31° 28′ 22″
		D	+11 46 44
		Latitude	+43° 15′ 6″

And this average agrees very closely with the latitude as determined from the single meridian observation. The reductions were taken from Rust's tables by simple interpolation, which is accurate enough for the purpose.

(*c*) Latitude by Pole Star.

On April 26th, in D.R. latitude 40° 48′ N. and longitude 4^h 56^m W. the corrected altitude of the Pole Star was found to be 39° 54′ at 8^h 25^m local m. t. Required the latitude.

Gr. m. t.	13^h	21^m	0^s
Acceleration	+	2	12
Sid. t. at Gr. m. n.	2	16	20
Gr. sidereal time	15^h	39^m	32^s
Longitude	4	56	
Local sidereal time	10^h	43^m	32^s
*'s R. A.	1	25	4
*'s Hour angle	9^h	18^m	28^s

True altitude	39° 54′
Correction	+ 54.6
Latitude	+40° 48′.6

The correction is taken by simple interpolation from Table IV of the *Nautical Almanac.*

CHAPTER IX

LONGITUDE

THE meridian of any place on the earth's surface is the great circle which passes through it and through the north and south poles. The meridians of all places thus converge, or run together, at the two poles. The actual distance in miles between the meridians of any two places varies with the latitude; the meridians are farthest apart at the equator and touch at the poles. Thus two meridians, which are exactly a degree apart, cross the equator at points distant one from the other, one degree or sixty (60) nautical miles. In the latitude of New York these same two meridians are only about 45 miles apart, while in the Polar Seas they approach to within 4 and 5 miles of one another. The statement that two such meridians are a certain number of miles apart, gives, therefore, no actual knowledge of their relative positions, unless the latitude of the point at which the distance is measured also be stated. But their relative positions are known as soon as their distance apart at the equator is given,

and this distance on the equator is called the difference of longitude. The longitude of a place, therefore, may be defined as the arc of the equator included between the meridian of the place and some standard or zero meridian. By common consent of all English-speaking peoples the meridian of Greenwich has been adopted as that meridian from which all longitudes shall be reckoned. Many other nations have acquiesced in this selection, and the charts of all countries, those of France, Spain, and Portugal excepted, are now referred to Greenwich. Hence the longitude of a place is its distance, measured on the equator, east or west from the meridian of Greenwich.

The problem of finding the longitude of a place resolves itself into that of finding the difference between the local time of the place and that of Greenwich at the same absolute instant. For, as we have seen, the earth rotates uniformly upon its axis, and this rotation carries the sun or other heavenly body at a uniform rate from east to west across meridian after meridian, and back again to the first meridian after the lapse of twenty-four (24) hours (solar or sidereal). In each and every hour the sun passes over one twenty-fourth part of the circumference of the earth, or over 15° of longitude. If, therefore, it takes one hour for the sun to pass from the meridian of one place to that of another, then the difference of longitude between the two places is exactly one twenty-fourth part of the circumference, or one

hour, or 15°. To determine difference of longitude, therefore, all that is necessary is to find the interval of time required for the sun to pass from the meridian of one place to that of the other; and this interval may be measured by an accurately running clock.

When the sun crosses the meridian at Greenwich it is noon, or zero hours, at Greenwich. When one hour later it crosses the meridian at a place 15° west of Greenwich, it will be noon at that place but one o'clock in the afternoon at Greenwich. The difference in local times between the two places will always be exactly one hour. The difference of longitudes, expressed in hours, is equal to the difference in local times at the same instant. And these local times may be expressed either in solar or in sidereal hours; it makes no difference, provided both times are given in the same units. The solar day, to be sure, is some four minutes longer than a sidereal day, and the sun appears to move more slowly across the heavens than do the stars. But the apparent motion of each body is due to the rotation of the earth, and this motion is uniform. It takes the sun one solar day or twenty-four solar hours to complete a circuit, it takes a star one sidereal day or twenty-four sidereal hours to pass once around the earth. And in one hour, therefore, solar and sidereal time respectively, the sun and a star will each pass over one twenty-fourth part of the circumference or over 15°. When the sun is observed, 15° of longitude are

measured by one hour of solar time, when a star is used the same 15° of longitude will be measured by one hour of sidereal time.

The problem of finding the longitude of a place is thus the problem of finding the respective local times of that place and of Greenwich at the same instant. It divides itself, therefore, into two distinct parts,

1. The finding of the local, or ship's time, at a given instant.

2. The finding of the Greenwich local time at the same instant.

The first part of this problem was fully treated of in the chapter on "Time and its Determination." At sea the ship's time at any instant is found from the altitude of the sun or of a star; the altitude being measured with a sextant. Such a sight gives the best results when the observation is made on the prime vertical, or when the body, at the time of observation, bears due east or west. An observation near the meridian is valueless for time determination, for in such positions the altitude changes very slowly and the time, as found from such an altitude, will be uncertain by many seconds.

The reason for the great superiority of an observation on the prime vertical is at once apparent when it is recalled that the line of position is always at right angles to the bearing of the object observed. When the sun bears east or west, the line of position runs north and south, and the longi-

tude of the ship is accurately fixed, although the latitude by dead reckoning may be in error by a large amount. In fact, if an observation could be made exactly on the prime vertical, the problem of finding the longitude would be reduced to and be identical with the problem of finding the Sumner line at that instant, just as the problem of finding the latitude is merely that of finding the Sumner line at noon.

The sun can rarely be observed on the prime vertical, for when it bears east or west, it is either below the horizon, or so close to it that the uncertainties of refraction more than counterbalance the advantages gained by making the observation on the prime vertical. But, while as a general rule it is thus impossible to obtain an observation in the best theoretical position, yet care should be exercised in getting a sight in as favourable a position as possible. The sight should be made at least three hours before or after noon, and to this fact is due the general custom of the "morning" and "afternoon" sight.

The second part of the problem, that of finding the Greenwich local time, is now extremely simple, and there is but one method in use, that of a chronometer or chronometers regulated to Greenwich time. Before the invention of the compensated balance for chronometers, the determination at sea of the Greenwich time was a most difficult operation, and elaborate and complicated astronomical methods were invented. Among

such methods, long since obsolete, may be mentioned eclipses of Jupiter's satellites and occultations of stars by the moon, and the celebrated method of "lunars."

It will be remembered that it was an investigation of the practicability of this method of lunars which led to the foundation of the Royal Observatory at Greenwich. It is the only astronomical method by which, on shipboard, the Greenwich time can be found with any approach to accuracy, yet it is so cumbersome and requires so much logarithmic work, that it may now be regarded as an interesting historical relic of no practical importance. The method depends upon the motion of the moon among the stars,—which motion has been so accurately tabulated by astronomers that they can predict, for many years in advance, the exact relative positions of the moon and stars for any moment of Greenwich time. These relative positions are published in the *Nautical Almanac* under the general heading of lunar distances, and give the distances of the moon from a number of the brighter stars and from the planets at intervals of three hours. The moon moves at an average rate of half a degree ($\frac{1}{2}°$) per hour, or of one minute ($1'$) of arc in each two minutes (2^m) of time. If, therefore, a star be directly in the moon's path, its lunar distance may be increasing or decreasing at this rate, but for all bodies situated above or below the moon's orbit the lunar distance will change less

rapidly. Thus in order to determine the Greenwich time to within twenty seconds (20^s), or the longitude to within $5'$, the distance of the moon from some one or more of the heavenly bodies must be measured to ten seconds ($10''$) of arc.

The method of lunars thus requires extreme accuracy of observation, an accuracy that can seldom or never be attained at sea. Not only is this high degree of accuracy in the observation itself necessary, but the reductions of the observation after it is made, are long and complicated and furnish many a chance for error. Owing to the comparative nearness of the moon, its parallax is large, amounting to some $57'$, and an actual observer on shipboard, therefore, sees the moon at a given instant in a widely different part of the heavens than would the theoretical observer at the centre of the earth for whom the distances in the *Nautical Almanac* are computed. It is the clearing of the measured distance from the effects of this parallax, and also from the effects of refraction, that cause the mathematical difficulties of the problem and lead to the long numerical solutions. For the seeker of the north pole, frozen fast into the ice floes and drifting for years across the polar seas, the method of lunars may yet offer advantages, but for the every-day navigator the method is now practically obsolete.

To-day the Greenwich time is always found from a chronometer, which has been rated before leaving port. Chronometers are now so carefully

made, that with proper care, they can be depended upon to give the Greenwich time to within thirty seconds at the end of six months after rating.

The time as read directly from the face of a chronometer is seldom, or never, the correct Greenwich time. To find therefrom the correct Greenwich time it is necessary to know the error of the chronometer at some one date and also its rate, or the amount that it gains or loses in one day. When these are known the Greenwich time corresponding to any subsequent chronometer reading can readily be found. For if the rate be constant, the error of the chronometer on Greenwich time will increase or decrease uniformly, and can be found by successive addition or by multiplication. If a certain chronometer at noon on Thursday, February 11th, be $6^m\ 31^s.8$ fast on Greenwich time, and it gains $1^s.8$ per day; then at noon on Friday it will be $6^m\ 33^s.6$ fast, on Saturday $6^m\ 35^s.4$, and ten days later it will have gained 18^s and will be $6^m\ 49^s.8$ fast. Thus the rate of a chronometer is the most important element; if the rate be constant the Greenwich time can be readily found, but if the rate varies from day to day the Greenwich time cannot be found with any degree of accuracy. The value of a chronometer depends therefore upon the constancy of its rate during long intervals of time. In a good chronometer this rate can be depended upon; in a poor chronometer

the rate varies, and such an instrument is worse than useless.

The great difficulty met with in the manufacture of chronometers is that the rate varies with changes in the temperature of the surrounding air. The balance wheel of a chronometer acts in a manner similar to the pendulum of a clock; a rise in temperature expands the metal and changes the rate at which the wheel vibrates. The compensated balance was introduced in 1765, when John Harrison invented and made the first successful marine chronometer. In this balance the rim of the wheel is divided into two arcs of about 180°: one end of each being fast to the opposite ends of a bar, which runs through the centre of the wheel and forms a diameter. On the free end of each arc are a number of adjustable weights, while the arcs themselves are formed of two metals, which under the action of heat, expand at different rates. Now as the temperature rises the metallic diameter expands and the wheel as a whole grows larger, but at the same time the outer metal of the rim expands faster than the inner. This causes the rim to bend and shortens its radius, thus bringing the weights nearer the centre. The general expansion of the wheel causes it to vibrate more slowly, whilst the bringing of the weights in towards the centre causes it to vibrate faster. By careful adjustment, therefore, the one effect can be made to offset the other, and the wheel to vibrate at a constant rate.

Although through the use of this compensating device chronometers can now be made which run with remarkable precision, yet the effects of temperature upon the rate cannot, even now, be wholly eliminated. The compensation may be nearly perfect for a small variation in temperature, but when an instrument is subjected to a wide range of varying temperatures, the rate is sure to suffer changes. On shipboard it is impossible to keep the chronometers in a constant temperature vault, such as are used for the standard clocks of an observatory. On long voyages and where the greatest precision is required, therefore, the rate of the chronometer should be determined for the different temperatures to which it is likely to be subjected. Its daily rates should be found for the average winter and summer temperatures, or at intervals of 10° from 40° to 100° Fahrenheit. With a maximum and minimum thermometer in the chronometer case, the actual average temperature of each day is recorded, and the rate corresponding to this average temperature should be used in calculating the chronometer error.

As a result of testing thousands of chronometers at the Liverpool Observatory, Mr. Hartnup discovered certain relations between the rate of a chronometer and the temperature. He found that every chronometer runs fastest (gains most or loses least) at some particular temperature, which can be determined by testing the chronometer at different temperatures. As the tempera-

ture varies, either increasing or decreasing from that at which the chronometer goes fastest, the chronometer runs more slowly: its rate varies as the square of the number of degrees of temperature from the temperature at which it runs the fastest. For example, if a chronometer runs fastest at a temperature of 75°, it will go slower as the temperature either rises above or falls below 75°; and its rate will be the same at 65° that it is at 85°, for both these temperatures differ from 75° by the same amount.

When a chronometer has been thoroughly tested and its temperature corrections determined, it can be relied upon to give most excellent results. The ordinary method of assuming the rate to be constant, regardless of temperature, may serve for short voyages, but does not give satisfactory results when the ship is exposed to widely varying temperatures.

All chronometers have slightly different rates at sea, from what they have on shore,—this being due, in all probability, to the jars and erratic motions to which they are subjected while on shipboard. To guard against accident and a sudden change in rate, a vessel should carry three chronometers, and each should be checked against the other. If there be two chronometers only and they disagree, it is impossible to tell which one is in error, but with three, if two agree and one differs the strong probability is that the error is in the one that differs. These chronometers

usually all keep mean solar time. It would, however, be of great advantage to have at least one sidereal chronometer, for such a chronometer would save much time and unnecessary computation in all star observations. The daily comparison of chronometers for checking rates can be made just as simply between a mean and a sidereal chronometer, as between two mean time instruments.

In working a sight for longitude there are three elements in any one of which an error may occur, and such error will introduce a corresponding, though not equal, error into the final result. These elements are the observed altitude, the assumed or D.R. latitude, and the chronometer time, and the effects of errors in each one of these will be investigated separately.

Error in Altitude

When the observed body is on the prime vertical, the line of position runs north and south, and an error of 1′ in the measured altitude will push this line of position one nautical mile to the east or west as the case may be. The error in the departure will be equal to the error in the altitude, but the resulting error in the longitude will depend upon the latitude, being greater the higher the latitude. In fact the error in longitude will be equal to the error in departure divided by the cosine of the latitude. Thus, except on the equa-

tor, an error in the measured altitude will always introduce a greater error in the resulting longitude.

When the body is observed off the prime vertical the error in the departure is greater than

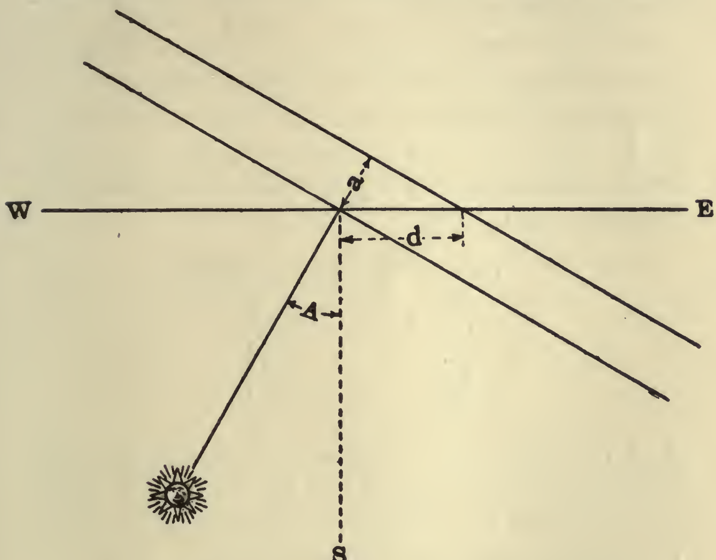

FIG. 18.—Error in Longitude due to Erroneous Altitude

the error in the altitude, and the departure error becomes greater and greater the farther the body is away from the prime vertical, and becomes practically infinite when the observation is made on the meridian. This is shown in the following figure, where *a* is the error in the altitude and *d* the corresponding error in the departure. As the azimuth becomes less and less, the two lines of position will swing around and become more

nearly parallel to the circle of latitude E W and *d* will become greater and greater. The resulting error in the longitude depends therefore upon the azimuth of the observed body and upon the latitude of the ship.

Lecky's Table D gives for different combinations of azimuth and latitude the error produced in the longitude by an error of 1′ in the observed altitude. From this table, for latitude 40°, the following errors corresponding to different azimuths, or bearings, are taken:

Azimuth	*Error in Longitude*
1°	74′.80
10°	7′.52
20°	3′.82
30°	2′.61
50°	1′.71
90°	1′.31

And this little table shows at a glance the advantage of observing on the prime vertical, and the practical impossibility of obtaining a satisfactory sight on or near the meridian.

Error in D.R. Latitude

Again the line of position is at right angles to the azimuth of the body, and will, therefore, run north and south when the body is on the prime vertical, and east and west when it is on the meridian. In the former case an error of 1′ will produce no error in the departure, while in the latter it will produce an infinite error. An intermediate case is shown in the accompanying

figure, where l is the error in the D.R. latitude and d the corresponding error in the departure. The error in the departure depends, therefore, upon the azimuth, while the resulting error in the longitude will also depend upon the latitude.

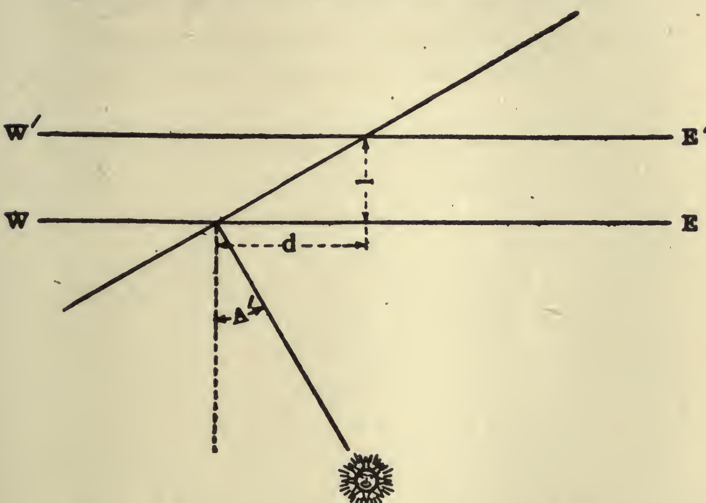

FIG. 19—Error in Longitude due to Erroneous Latitude

Lecky's Table C gives for different combinations of azimuth and latitude, the error produced in the longitude by an error of 1′ in the D.R. latitude. From this table the following extracts are taken for latitude 40°:

Azimuth	*Error in Longitude*
1°	74′.79
10°	7′.40
20°	3′.59
30°	2′.26
50°	1′.09
90°	0 .00

This again shows the immense superiority of observations made on or near the prime vertical, in contradistinction to those made on or near the meridian. In latitude 40° if the observation be made within 1° of the meridian, an error of only one mile (1′) in the latitude by D.R. would introduce into the longitude derived from a perfect sight an error of nearly one degree and a quarter (74′.8).

This Table C of Lecky's is extremely valuable in the every day work of carrying a morning sight forward to noon. When the morning longitude sight is made the correct latitude is unknown. In working out the sight the latitude as brought up by dead reckoning from the former noon is used, and this D.R. latitude may be in error by some miles, and consequently the longitude, as found from the sight, will be in error. If, however, a subsequent latitude sight is obtained at noon, the morning longitude can be carried forward to noon by allowing for the ship's run in the interval between sights. But the noon longitude thus obtained will be in error, because the longitude derived from the morning sight was itself in error.

The noon sight will give the true latitude and thus show the error in D.R. latitude used in working up the morning sight. From this error can be found by Lecky's table the corresponding error in the longitude, and thus the noon longitude can be immediately corrected, without the neces-

sity of re-working the morning sight. The following example will probably make this use of the tables more clear.

In D.R. latitude 39° 51¼′ N. a morning sight for longitude was made, the bearing of the sun being S. 68° E. When worked up the sight gave the longitude as 52° 51′ W. From the morning sight until noon the ship ran due east, changing its longitude by 34½′, so that at noon the D.R. longitude was 52° 16½′ W. The noon sight gave 39° 41′ N. as the true latitude, showing an error of −10¼′ in the assumed latitude with which the morning sight was worked up. Now Lecky's table shows that in latitude 39° and azimuth 68°, an error of 1′ in the latitude will produce an error of only 0′.52 in the deduced longitude. Hence the error of 10¼′ in the D. R. latitude gave an error of 10¼ × 0′.52 or 5¼′ in the longitude. The true longitude at noon, therefore, was 52° 21¾′.

This same result of course could be obtained by re-working the morning sight with the corrected latitude, but this would take far more time than the above simple method.

Error in Chronometer

An unknown error in the chronometer will produce an equal error in the derived longitude, and this will be true no matter what the latitude of the ship, or the azimuth of the body observed. The chronometer error merely shifts the local meridian to the east or west by a like amount.

NOTES AND PRACTICAL APPLICATIONS

1. *Formulas for Longitude.*

Measure the altitude of any body, sun, star, or planet, when it is on the prime vertical, or as near the prime vertical as practicable, and note the time on a chronometer regulated to Greenwich mean time. Correct this altitude for dip, refraction, and semi-diameter, if necessary, and from this corrected altitude, together with the latitude by account, find the hour angle by the formula,

$$\sin^2 \tfrac{1}{2}\, t = \frac{\cos S \sin (S-h)}{\cos L \sin P}$$

where,

$$S = \tfrac{1}{2}\,(h+P+L)$$

Then:

1st for the sun.

Correct the observed chronometer time for chronometer error and for the equation of time, thus finding the Greenwich apparent time. The difference between this and the above found hour angle is the longitude of the ship, at the moment the observation was made.

2nd for a star or planet.

If a *mean time* chronometer be used, correct the observed time for chronometer error, thus finding

the Greenwich mean time. Convert this into sidereal time by adding to it the sidereal acceleration (found by Table III, *N. A.*) and the sidereal time at Greenwich mean noon (from the *Nautical Almanac*); the result is the Greenwich sidereal time.

To or from the right ascension of the body add (if west) or subtract (if east) the hour angle as computed above and the result is the ship's sidereal time. The difference between these two times (Greenwich and ship's) is the required longitude.

If a *sidereal* chronometer be used, correct the observed time for chronometer error, thus directly finding the Greenwich sidereal time. Then to or from the right ascension of the body add (if west) or subtract (if east) the hour angle as above computed and the result is the ship's sidereal time. The difference between these two times (Greenwich and ship's) is the required longitude.

2. *Practical Examples.*

On May 27th, in D.R. latitude 40° 55′ N., the altitude of the sun's lower limb was 34° 4′ at $8^h\ 53^m\ 40^s$ Greenwich mean time; index correction, + 1′ 10″; height of eye, 10 feet. Required the longitude.

Obsd. altitude	34° 4′
Index	+ 1 10″
Dip	− 3 6
Tables IV & V	+ 14 51
h	34° 16′ 55″
L	40 55
P	68 39 35
	2)143° 51′ 30″
S	71° 55′ 45″
S − *h*	37 38 50

Declination.

At G. mean noon	21° 16′ 39″ N.
Corr. for $8^h.95$	+ 3 46
D	21° 20′ 25″ N.

Equation of Time.

At G. mean noon	$3^m\ 6^s$
Corr. for $8^h.95$	− 2.5
Equ. (add to mean)	$3^m\ 3^s.5$

	cos S	9.49163
	sin $(S-h)$	9.78589
	sec L	0.12167
	cosec P	0.03085
	haversine	9.43004
Hour angle		4^h 10^m $1^s.5$
Chronometer time		8^h 53^m 40^s
Equation of time		3 3.5
Greenwich app. time		8^h 56^m $43^s.5$
Ship's " "		4 10 1.5
Longitude		4^h 46^m 42^s.
		71° 40½′

Or the longitude of the ship is 71° 40½′ west of Greenwich. The azimuth of the sun was N. 91° W., or the sun was practically on the prime vertical, and hence in the best position for determining the longitude.

Had the sight been worked with a latitude of 40°, or with an error of 55′, the resulting longitude would have been 4^h 46^m $42^s.4$, or less than 0′.1 different from that obtained with the D.R. latitude. This shows the great advantage of taking a longitude sight on, or near, the prime vertical. For latitude, the navigator is accustomed to wait for the meridian passage of the sun or star: why should he not wait for prime vertical passage, when he desires the longitude?

CHAPTER X

THE CAUSE OF THE TIDES

THE tides are the regular periodic changes in the level of the sea caused by the attractions of the sun and moon. The waters rise and cover the rocks and shoals along the coast, then sink back, leaving the reefs and beaches again bare; and this rising and falling of the waters is repeated day after day and year after year at regular recurring intervals of time. In general the interval from one high water to the next is a little over twelve hours, so that at nearly every point of the coast there are two tides each day. But the tides of one place may differ radically from those of another.

On the coast of New Jersey the average rise and fall is about four feet; on the coast of Maine it varies from 9 to 20 feet; while in Minas Basin, at the head of the Bay of Fundy, it is about 43 feet, the rise and fall of the spring tides here being 50 feet. Here occur the greatest of all existing tides. At Galveston, Texas, and at St. Michael, Alaska, there is, in general, but one tide in every

twenty-four hours, while at Port Townsend on certain days in each month the water will remain very nearly stationary and at a high level for eight and ten hours, then suddenly fall many feet.

With few exceptions, however, the tides of all localities are characterised by two noticeable peculiarities: a daily retardation in the time of high water, and a monthly variation in the height of the tides. The average interval from one high water to the next is about twelve hours and twenty-five minutes, so that, each day, the time of the corresponding high water is apparently delayed some fifty minutes. If it be high water to-day at nine o'clock in the morning, it will be high water to-morrow at ten minutes before ten, and at nineteen minutes before eleven on the day after. This retardation is not always the same, it varies a little during each month; on some days it is a few minutes greater and on others a few minutes less than the average. Not only does the retardation thus show periodic changes during each month, but so also are found noticeable monthly variations in the heights of the tides. The rise from low to high water is called the "range" of the tide, and the range is greatest at the times of new and full moon, and least when the moon is in quadrature. At times when the range is the greatest, the waters rise highest and fall lowest; and these, the greatest tides of the month, are called "spring" tides. The tides when the range is the smallest are called "neaps."

PLATE IX

High and Low Tide in the Petitcodiac River

LOW TIDE

A 2500-ton Steamer Loading at Wharf on the Petitcodiac River at the Head of the Bay of Fundy

HIGH TIDE

A Steamer at the Same Wharf Loading Plaster

Now these two characteristics of the tide clearly point to the sun and moon as ultimate causes. The average daily retardation of the tide is identical with that of the moon; the interval between two successive passages of the moon over any one meridian being twenty-four hours and fifty minutes. And again the variations in the retardation and in the range of the tides depend upon the phase of the moon, upon its position relative to the sun. When these two bodies are in such portions of the heavens that their disturbing attractions for bodies upon the earth are added together, the tides are the highest; when they are so situated that the disturbing attraction of the sun is opposed to that of the moon, then the range of the tides is the smallest. Further, the distance of the moon from the earth varies greatly during each month, and the tides change with this varying distance. When the moon is nearest the earth, the tides are nearly thirty per cent. of their average height greater than when she is farthest off. The height, or rather the range, of the tides thus depends primarily upon the phase of the moon and secondarily upon the distance of the moon from the earth. The highest tides of all occur when these two causes act together, or when new or full moon happens when the moon is nearest the earth, or in perigee.

The special characteristics of the tide of a given place are best shown on a tidal curve, similar to that in Fig. 20. This curve, which is typical

of the Middle Atlantic coast, was recorded during four weeks of August, 1906, on an automatic gauge at Shelter Island, N. Y. Such a tidal gauge consists of two essential parts, a float, which rises and falls with the water, and a recording apparatus. The surface of the water is constantly ruffled by waves, which keep an ordinary buoy or float in constant agitation and which completely mask the effect of the tide. In order to secure smooth water for the float, a well or tank is sunk near the beach line, and this well or tank is connected with deep water by a pipe of small diameter ending in a perforated rose or nozzle. The sea-water flows freely back and forth through this pipe line, but the disturbing effect of the waves is effectually destroyed and the water in the tank is always at the same level as that of the open sea. In this tank is a metal can-shaped float or buoy, which rises and falls with the changes in sea-level. This float is suspended from a wire, which leads through a system of levers or wheels to the pencil of the recording device. This pencil rises and falls with the float, but for convenience its motion is reduced in some proportion; the pencil of the Shelter Island gauge moves, for example, one inch for every foot rise or fall of the float. The paper against which the pencil rests is drawn forward by clockwork so that a continuous curve is drawn. Thus is obtained a permanent record which shows the height of the sea at any moment of the day or night.

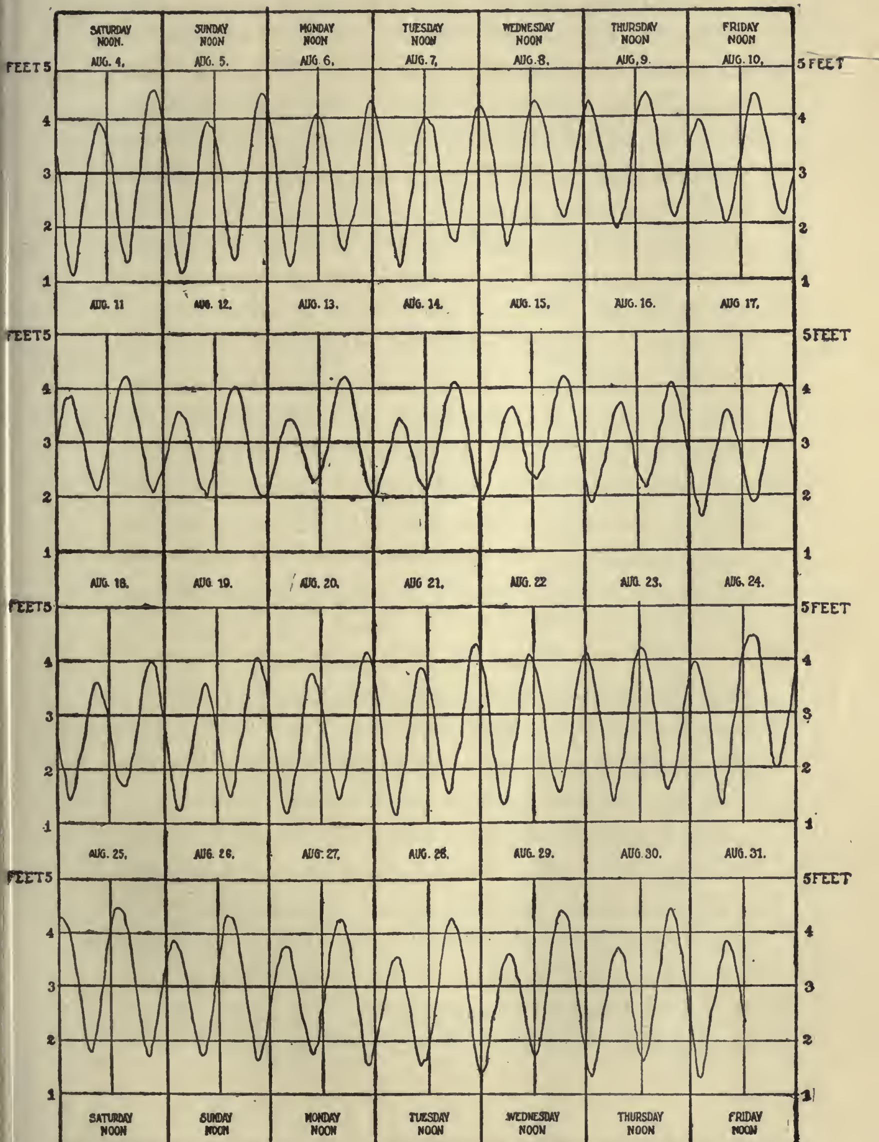

FIG. 20.—Shelter Island Tide Curves for August, 1909

In the diagram the curve is broken up into four parts, each part representing the tidal record for one week, from noon of one Friday to noon of the next Friday. The curves for the corresponding days of the week are thus found in the same vertical line. The vertical heights give the level of the water at any instant on an arbitrary scale of feet; the horizontal scale shows the time, the hours of the day and night. The first striking peculiarity of these tidal curves is the great disparity between the heights of the two tides each day. In nearly every instance the afternoon or evening tide is from six inches to a foot higher than the morning tide. This is the diurnal irregularity, and this irregularity will be specifically discussed later. The daily retardation of the tide is also well exhibited. On August 4th it was high tide at 9:45 A.M.; on the 5th at 10:30; on the 6th at 11:15; and on the 7th at 12 o'clock noon. By the 18th the retardation amounted to nearly twelve hours, the times of high and low water being very nearly the same as on the 4th; but the high water, which occurred at 9:45 in the morning of the 4th, on the 18th came at 10 o'clock in the evening, a total retardation of twelve hours and fifteen minutes in fourteen days. This gives an average daily retardation of fifty-two (52) minutes, agreeing very closely with the corresponding quantity for the moon. Thus, so far as the times of high and low water are concerned, the tides repeat themselves very closely every fortnight.

The character of the tides varies conspicuously during the month. On August 4th to 6th the range of the tide was the greatest; the largest variation in level between consecutive high and low waters being 3.5 feet between the afternoon tide of August 4th and the morning tide of the 5th. The smallest variation occurred in the morning tide of the 13th, when the difference between high and low water was only 1.1 feet, less than one third that of the 4th. From this time on the range increased until the 21st, when it was a little more than three feet, and then it began again to diminish, reaching a minimum of 1.8 feet on the 29th. The moon was full on August 4th, new on the 19th, and the first and last quarters fell on the 26th and 11th August respectively. The relation between the spring and neap tides and the phases of the moon is thus clearly brought out. Still further the neap tides of August 12th and 13th were lower than those of the 28th and 29th, and the reason of this is clear, for the almanac shows that on August 12th the moon was at its greatest distance from the earth, whilst on August 26th it approached nearer our planet than at any other time during the month.

In Fig. 21 are reproduced, from the report of the U. S. Coast & Geodetic Survey for 1897, the tidal curves for St. Michael, Port Townsend, and Havre. The blunted tops of the curves for Havre show the long stand of the tide as compared with the short stand on the Atlantic coast.

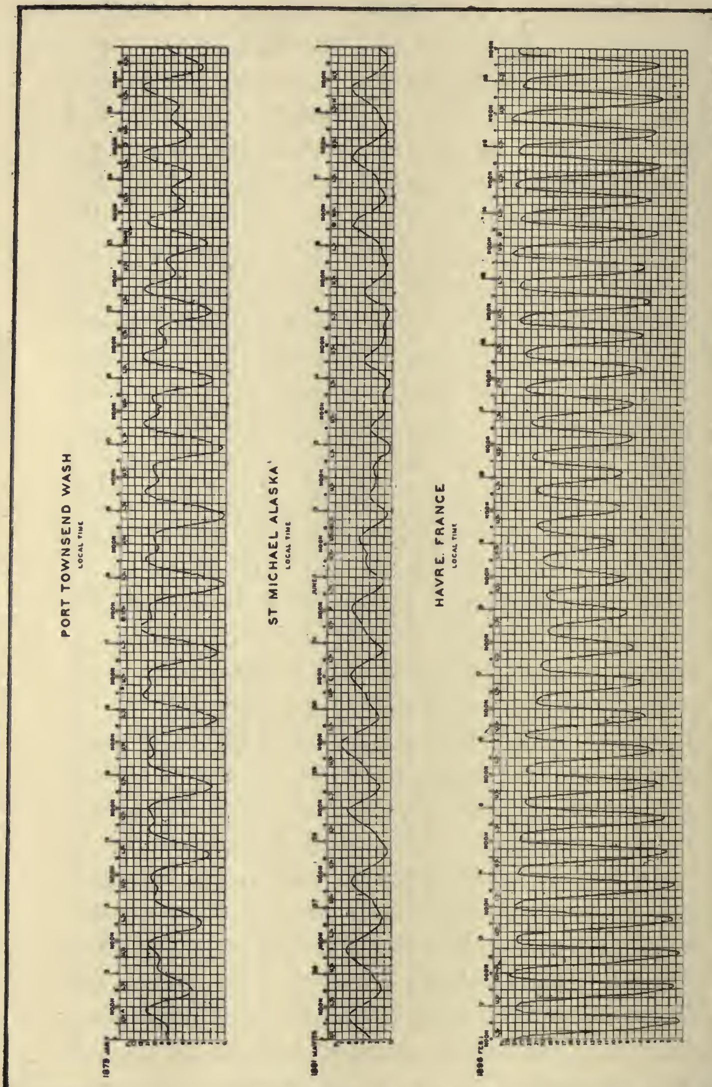

FIG. 21.—Tide Curves for Port Townsend, St. Michael, and Havre—(U. S. Coast and Geodetic Survey).

At Shelter Island the tide rises to its maximum, and begins almost immediately to fall, the average stand recorded during the month of August being only a few minutes, while at Havre the average stand is over three hours. At Port Townsend the tide is double-headed and remains high for many hours.

The intimate connection between the tides and the moon has been recognised for many centuries, but not until the time of Newton was the reason for this connection known. In the *Principia* he shows that the tides are a direct and necessary consequence of the law of gravitation. According to the Newtonian law the moon attracts each and every particle of matter in and around the earth, and the strength of this attraction varies inversely as the square of the particle's distance from the moon. Now all those particles which are united in the solid or rigid portions of the earth form a great sphere, or globe, eight thousand miles in diameter. This globe is attracted toward the moon as a whole, the strength of the attraction depending upon the average distance of all the constituent particles, and this average distance is that of the centre of the earth. A particle on the surface of the earth directly under the moon will be attracted more strongly than is the earth as a whole, for it is nearer the moon than the average particle at the centre. The moon tends to draw such a surface particle away from the earth, but this lifting force is very small compared

with the whole attraction of the earth, and the action of the moon simply lessens the weight of such a surface body to a very small extent.

The distance from the moon to the centre of the earth is very nearly sixty times the radius of the earth, and hence the attraction of the moon upon the earth will be proportional to $(\frac{1}{60})^2$. The surface particle directly under the moon is one radius, or four thousand miles, nearer the moon than the earth's centre, and hence the attraction of the moon for it will be measured by $(\frac{1}{59})^2$. The difference between these two quantities, or $(\frac{1}{59})^2-(\frac{1}{60})^2$, measures the lifting force of the moon for the particle in question. Reducing these fractions to decimals, and taking the difference, the result is 0.000,009,496. Now at equal distances the pull of the moon is only one eightieth $(\frac{1}{80})$ that of the earth; therefore, in order to compare this lifting force of the moon with gravity the above decimal must be multiplied by $(\frac{1}{80})$. The result of this multiplication is 0.000,000,1187, or in vulgar fractions, $(\frac{1}{8,424,000})$. That is, because of the existence of the moon a 4000-ton ocean steamer loses one pound of its weight when that body is directly overhead.

This disturbing action is not confined to particles at A, directly under the moon, but affects, to a greater or less degree, every portion of the earth's surface. At B, directly opposite the moon, there is a lifting force almost exactly equal to that at A. The attraction of the moon for the

average particle of the earth at C is greater than that for a particle at B, for B is at a greater distance from the moon than is C. The moon tends to pull the earth as a whole away from the particle

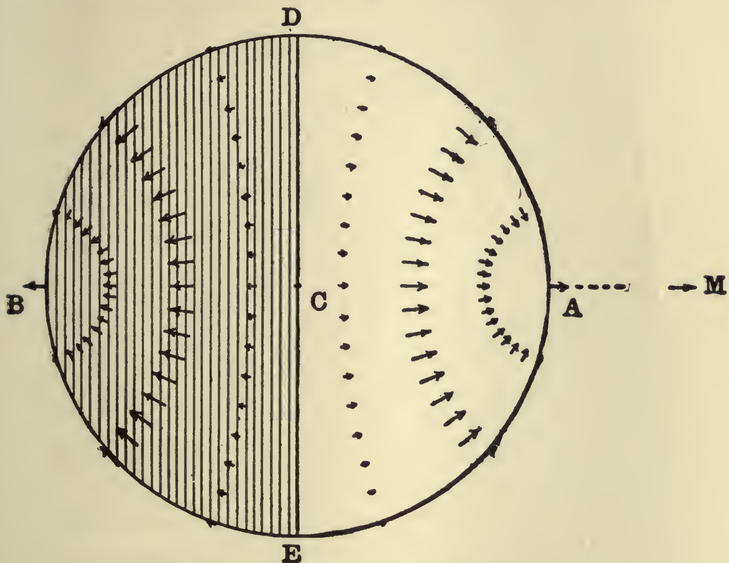

FIG. 22.—Tide-generating Forces

B, thus diminishing the force of gravity and causing a lifting force. At D and E and all points where the moon would be on the horizon, the effect of the attraction is to increase gravity, to pull bodies toward the centre of the earth. At D, a 4000-ton ship would gain about one half a pound in weight, or would weigh nearly a pound and a half more than when at A. At intermediate points on the surface the disturbing force is neither

directly up nor down, but is partly vertical and partly horizontal. On two small circles, however, about half-way between A and D, and D and B, respectively, the force is entirely horizontal and tends to move particles along the earth's surface.

In waters having depths no greater than existing ocean depths the vertical force has little or nothing to do with forming the tides; it may slightly alter the weight of the water, but it cannot lift it. In an ocean of uniform depth and density, the vertical force, directly under the moon, would decrease the weight of the water by the $\frac{1}{8,424,000}$ part of itself. Thus in a sea three miles deep the greatest variation in weight or pressure due to the variation in the vertical force could never be greater than the weight of a layer of water $\frac{3}{100}$ of an inch in thickness over the area considered. Furthermore the vertical force acts perpendicularly to the free surface of the water, and it cannot make the water particles move horizontally or sideways. This vertical force is, therefore, of no importance in the tidal theory—the tides are produced by the horizontal force.

This horizontal force is of approximately the same magnitude as the vertical, but its distribution over the earth's surface is quite different. At the points where the moon is directly overhead and directly under foot, the horizontal force vanishes, and it is just at these two points where the vertical force is the greatest. Again, on the

circle which passes through all those points at which the moon is just rising or setting there is no horizontal force. But at every other point of the hemisphere illuminated by the moon there is a horizontal force directed toward the sub-lunar point; at every point on the dark hemisphere there is a horizontal force directed toward the point at which the moon is directly under foot. These horizontal forces are shown by the arrows in Fig. 22.

Now the water of the oceans is mobile and a very minute force will cause a particle to move horizontally, whereas a large force is necessary to overcome gravity and to lift the particle directly upwards. The action of the horizontal tide-generating forces is to move the particles along a horizontal path towards the sub-lunar points. In order to see just how an individual particle is affected, let us consider a particle on the equator, with the moon, also on the equator, just rising in the east. At this instant there is no horizontal force acting upon the particle; there is, however, a very minute downward tendency due to the vertical force. As the moon rises above the horizon the particle is drawn toward the east, at first gently, then more and more strongly, until when the moon reaches an altitude of about 45° the eastward pull on the particle is the strongest. As the moon rises still higher the eastward pull continues, but gradually diminishes in strength, finally vanishing when the moon is

directly overhead. As the moon sets in the west, the particle is drawn to the west, being drawn most strongly when the moon is half-way between the zenith and the horizon. When the moon is setting on the western horizon, the horizontal force vanishes and the conditions become exactly as they were at the start.

All the time from moonrise to transit the moon is urging the particle to the eastward, and its eastward velocity, circumstances permitting, is therefore continually increasing. At transit this eastward acceleration ceases, and at this moment the particle will have its maximum eastward velocity. From this moment on, as the moon sets in the west, the acceleration is westward. This westward acceleration first diminishes the eastward motion of the particle, brings it to rest, and then imparts a faster and faster westward motion, which reaches its maximum as the moon passes below the horizon. Thus the particle is moving fastest to the eastward at lunar noon and midnight, and fastest to the westward at moonrise and moonset. The moon through the agency of the horizontal forces thus sets the particle of water vibrating in what is practically a horizontal path. The particle oscillates back and forth in this path, making one complete oscillation in twelve lunar hours.

Water is sensibly incompressible; hence unless all the particles move in unison, and at the same speed, the surface must rise and fall and be dis-

torted into waves. But, under the action of the tide-generating forces, the particles do not move in unison. They all vibrate in the same way and in the same period, twelve lunar hours, but the phases of their vibrations are different. The moon passes over particles to the east earlier than it does over those to the west; hence, while some particles are moving most rapidly to the eastward, others to the east of them are practically at rest, while particles still farther away are moving with their greatest speed toward the west. At certain points, therefore, the ocean must rise and fall, and the paths of the separate particles be transformed into greatly elongated ellipses. In an east-and-west canal extending around the earth, or in an ocean covering the earth uniformly, the wave will have a fixed position relative to the moon, and the wave form will therefore travel westward around the earth. From the simplest consideration of wave motion it is evident that, in such a canal, the crest must be at that point at which the particles are moving with their greatest velocity towards the west, the trough where the particles are moving at the maximum speed towards the east. High water will, therefore, occur when the moon is on the horizon, low water when it is on the meridian.

If, therefore, the equatorial regions of the earth were covered by a shallow ocean and the declination of the moon were always zero, an observer at the equator would have two tides

each lunar day; the high tides occurring when the moon was just rising and setting; the low tides, when the moon was overhead and under foot. The tidal wave would be 12,000 statute miles from crest to crest, and slightly less than two (2) feet in height.

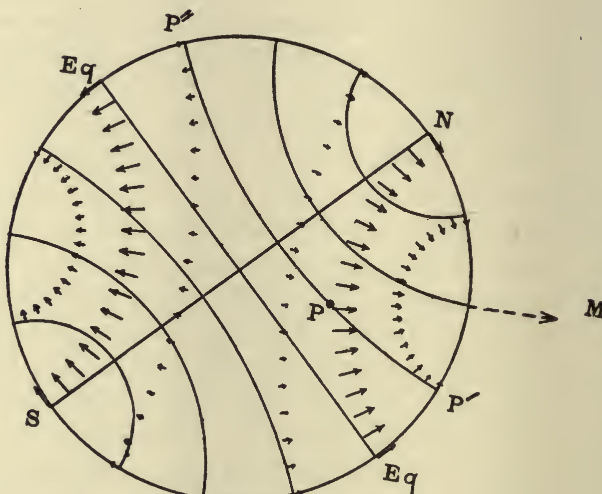

FIG. 23.—The Diurnal Variation

When the moon is not on the equator, the tide-generating forces are not so simple as those just considered. Fig. 23 represents the horizontal tide-generating force at various points on the surface, when the moon is at some distance north of the equator. The arrows point in the direction in which the force acts, and their lengths indicate

roughly its varying intensity. Now as the earth rotates on its axis the moon passes along a circle of latitude, and carries with it the system of arrows, which thus sweeps over the surface of the earth. Any particle situated at P, for example, is thus subjected in succession to the forces represented by all the arrows lying in the same latitude as P. Now at two points on this circle of latitude, 180° apart in longitude, as P′ and P″, the forces are not the same either in magnitude or in direction. Hence as the moon revolves around the earth, the forces at P will not be exactly repeated every twelve lunar hours—it will take a full lunar day before the forces at P become again the same in direction and in intensity. From moonrise to moonset the force goes through a certain definite series of changes, from moonset to moonrise through another definite series of changes, returning at moonrise to the initial condition. Thus during a tidal day there are two distinct periods—one, the full day at the end of which all the phenomena repeat themselves; the other, the half day.

This double periodicity in the generating forces gives rise to the difference in the heights of the morning and evening tides. The tidal wave is compounded of two distinct component waves, with periods equal respectively to 24 and 12 hours—these waves are known as the "diurnal" and the "semi-diurnal" components.

The semi-diurnal wave is generally the more

important of the two. Its height at any given place is almost constant, for it is sensibly independent of the moon's declination. The diurnal component, on the other hand, varies in height with the ever-changing declination of the moon, and disappears when the moon is on the equator. Twice each month, therefore, the diurnal wave vanishes and on these dates all places have pure semi-diurnal tides—the two successive tides are exactly alike.

The ratio between the heights of the two component tides and the relative positions of their crests depends upon the latitude of the place and upon local conditions of the coast, depth of water, etc. And these variations govern and cause the difference in the *type* of the tide.

Effect of Forces in Producing Tides

So far we have noted only that the moon imparts periodic (12-hour and 24-hour) vibrations to the particles of water. The character of the resulting wave has not been discussed, except for an hypothetical equatorial canal of uniform depth. In reality there is no such canal, and the conclusions drawn from its consideration are of little practical importance. The waters of the earth are broken up into many lakes, channels, and oceans of irregular shapes and varying sizes, and it is of the highest importance to consider the action of the tide-producing forces upon such bodies of

water. These effects are widely different upon bodies of water of different sizes and depths; especially are they different on lakes and small enclosed seas from what they are on the broad, deep oceans. And this difference may be traced to the varying periods required for a free wave to travel back and forth across the comparatively short sea and the wide ocean. Such a wave, if long as compared to the depth of the water, will travel at a speed depending solely upon the depth. If the water be 200 feet deep, the wave will move at a rate of about 55 statute miles an hour; if the water be shallower, the speed will be less; if deeper, greater. In deep seas and oceans the speed of a long free wave is from 400 to 500 miles an hour.

Each body of water, large and small, has its own period of vibration, the length of time required for a long free wave to travel forward and back across its surface. Take a rectangular shallow tray of water, for example. If one end be suddenly raised and immediately lowered again, a long wave will be started, which runs to the other end of the tray, is reflected, and returns to the starting point. The wave again sets out and continues to run back and forth, growing smaller and smaller until the water gradually returns to rest. The length of time taken by the wave in one complete journey to and fro across the tray is the vibration period, and this period depends upon the length of the tray and the depth of the water. In a lake

100 miles long and 200 feet in depth, the wave, travelling at about 55 miles per hour, would require something less than four hours to complete its journey. The north Atlantic Ocean is some three thousand miles wide, yet it is of such an average depth that a free wave travels at a rate of some 500 miles per hour. The wave requires, therefore, twelve hours to pass from Europe to America and return, and thus its complete period of oscillation is some twelve hours.

Now the tide-generating forces disturb the water in a lake or the ocean and give rise to a wave, and exactly twelve hours later this disturbance is repeated. The waters are thus subjected to regular periodic shocks, and the intervals between these shocks are independent of the size or shape of lake or ocean—the intervals depend solely upon the motion of the moon. The waters of a lake, as we have seen, will, when left to themselves, oscillate in a definite period—a period fixed by the shape, size, and depth of the lake. The lake naturally oscillates in one period; the tide-generating forces tend to make it oscillate in a different period. What is the result? How will the waters of the lake act, how will they oscillate?

This is answered by considering for a moment the general dynamical principle involved. If any system, naturally oscillating in a certain free period, be acted upon by a force varying with a different period, then the system will always

vibrate in a period identical with that in which the force acts—the free period being completely destroyed—but the character of the oscillation will be different, depending upon whether the free period be greater or less than the forced period. When the period of the force is greater than the free period, then the oscillations of the system agree with the variations of the force; when the period of the force is less than the free period, then the resulting oscillations of the system are inverted—being greatest when the force is least, and least when the force is greatest. This principle is probably best shown with a pendulum. Now the length of time required for a pendulum to swing back and forth depends solely upon its length —a string 39 inches long, with a heavy weight at the end, will swing in exactly one second. If the string be held in the hand and moved regularly to and fro through a short distance, the pendulum will be started swinging; the interval required for the hand to move back and forth is the forced period; the free period is one second. If the hand move very slowly, taking five seconds for each motion, then the pendulum will keep pace with the hand: when the hand is farthest to the right, so also is the pendulum. The free period in which the pendulum would naturally swing has been destroyed, and the pendulum oscillates with the period of the disturbing force, and swings in unison with the force.

Next move the hand rapidly back and forth in,

say, one half-second. The pendulum now oscillates very quickly, but whenever the hand is to the right, the bob of the pendulum is to the left, and when the hand is to the right, the pendulum is to the left. The free period of the pendulum has

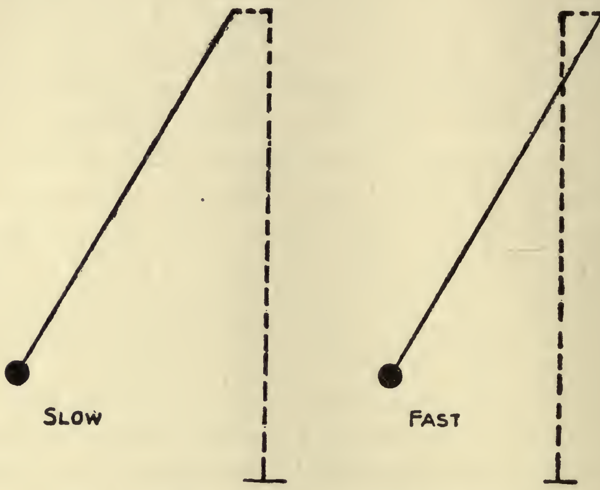

FIG. 24.—The Forced Vibrations of a Pendulum

again been destroyed—it swings with the period of the disturbing force, but its motion is inverted.

Now apply this principle to a narrow lake or an enclosed sea at the equator, the natural period of vibration of which is much less than twelve hours. The forced period is greater than the free period, and hence, under the action of the tide-generating forces, the lake will oscillate in a period of twelve hours, and the oscillations will be in

unison with the force. When the moon is rising in the east, the horizontal force is towards the east, and the water of the lake is drawn to the east; the eastward force is the greatest when the moon is half-way between the horizon and the zenith, and at this time, or at three hours before lunar noon, it will be high tide at the eastern end of the lake. Six hours later, when the moon is sinking toward the west, the force will be towards the west and will have its maximum westward strength. As the water moves in unison with the force, it will be high water at the western end of the lake at this time, or three hours after lunar noon. The waters of the lake thus rock backward and forward about a north and south no-tide line in a period of twelve hours, it being high water at the eastern and western ends of the lake at alternate periods of six hours.

The approximate height of the tides in such a lake can be found readily. For in the oscillations the surface of the water is always normal to the direction which a plumb-line takes under the disturbing action of the tidal forces. At its maximum the horizontal force is only about $\frac{1}{11,232,000}$ part of gravity, and the resulting deviation of the plumb-line is only $0''.018$ from its mean position. The surface of the lake, therefore, rocks backward and forward through this minute angle, the radius of the oscillating arm being one half the length of the lake. The total range of the tide from high to low water at one

end of a lake 120 statute miles long would be only a little more than $\frac{2}{3}$ of an inch.

An example of such an equilibrium tide is presented by that part of the Mediterranean Sea to the east of Sicily. The passage between Sicily and Tunis is so narrow that each half of the Mediterranean may be considered by itself, as a closed sea. The eastern portion is some 1100 statute miles long and of an average depth considerably over 1000 fathoms, and its period of free oscillation would thus be approximately three hours. As this period is much less than the period of the tidal forces, the surface of the sea should rock back and forth about a central line, it being high water at the eastern end, or on the coast of Syria, at the time it is low water at Sicily; and vice versa, when it is high water at Sicily, it should be low water at Syria. And further, these high and low waters should occur three hours before and after the moon's transit over the middle of the sea. The range of the tide should vary from about half a foot at the ends to nothing at the centre. These conclusions agree approximately with the observations. At Malta the corrected establishment of the port is $3^h\ 12^m$, while at Alexandria and at Port Said it is about 10^h. At these places the spring range of the tides is a little less than one foot, while at Candia and Crete the tides are extremely small.

Now let us consider what would happen in a long lake or canal, one so long that it would require

over twelve hours for a free wave to travel back and forth across its surface. In this case the free period is longer than the forced period and the waters will oscillate with the period of the force, but the oscillations will be inverted with respect to the force. When the eastward force is the greatest, when the moon draws the plumb-line most strongly to the east, then will it be high water at the western end of the lake; when the moon draws the plumb-line to the west, it will be high water at the east. The oscillations will be of the same character as in the small lake, but the times of high and low water will be inverted. The surface of the water is not normal to the plumb-line. If the canal were exactly one half wave-length long, it would be high water at the eastern end when the moon crossed the meridian of the middle point of the canal, and high water at the western end six lunar hours later. If the canal were so long as to completely encircle the equator, then in the hemisphere towards the moon the waters in the eastern portion and the waters in the western portion would slope downward toward the point directly under the moon. It would be low tide at the point where the moon was overhead, high tide at the points where the moon was just rising and setting. Now this is exactly what was found from a consideration of the wave produced by the oscillations of the individual particles of water. The two ways of looking at the tidal

disturbances in an hypothetical equatorial canal give thus identical results. If the earth were surrounded by a shallow ocean, not more than three or four miles deep, the tides at the equator would be inverted. It would be low tide directly under the moon, where one would naturally look for a high tide.

If the natural or free period of oscillation of the lake or ocean be exactly equal to the period of the tidal forces, what then? The oscillations of the water would grow greater and greater without limit; each succeeding tide would be higher and higher, until the waters would so overflow the banks of the lake or ocean as to alter the character of the oscillation. That is, long before this disruption could take place, the waves would be so large that forces and conditions, hitherto neglected, would be brought into play and modify the simple results. When the oscillation becomes large, the resistance offered to the motion of the water particles would eventually become sufficient for preventing the tidal forces from further augmenting the oscillation. While thus the tides can never really become infinite even if at the start the two periods are exactly identical, yet they may become very large.

When the period of free oscillation of an ocean is very nearly equal to the period of the tidal forces, the tides may, for this reason, be considerably larger than they would if the two periods were radically different. The north

Atlantic from Europe to America has a period of about twelve lunar hours for a complete oscillation, and to this is traced the abnormally large semi-diurnal tides.

So far we have discussed simple oscillations, or standing waves, in comparatively narrow canal-like bodies of water. But the lakes and oceans of the earth are irregular in shape and of varying depth. They are all comparatively wide, and this very width introduces complications. The vibrations of a wide lake are extremely complex as compared to those of a narrow canal of the same length. In fact a complete mathematical solution of the character of the vibrations of such a lake is impossible. But the general type of the oscillation is known. Under the action of the tide-generating forces the surface of a circular lake, for example, rocks about a "no-tide" point. Imagine a circular disc of cardboard, such as a dry compass card, resting on a central pivot and so weighted at one point that the surface is not quite horizontal. The highest point of the circumference will represent high water, and the lowest point low water. Now turn the card slowly around the pivot and the line of high and low waters will successively take up all directions; the high point always being directly opposite the low point. When the high point is to the eastward of the pivot, the low point will be to the westward; when the high is to the north, the low will be to the south. Further, the height above

the pivot of any intermediate point on the line of high and low water will depend upon the distance of that point from the centre of the card. A point half-way out toward the circumference will be only one half as high above the pivot as the point on the edge. Thus, in this imaginary illustration, it would be high water at all points due east of the pivotal, or no-tide, point at the same instant, but the height of the tide would increase with the distance from the central point. Now a line, which connects all these points at which it is high water at the same instant, is called a "co-tidal" line, and on our imaginary card all the co-tidal lines would be straight, and would radiate from the pivot, like the spokes of a wheel; the lines of equal rise and fall would be concentric circles, with the pivot as common centre.

In the case of a deep lake the tidal oscillation of its surface is very similar to the rotating card. As the size of the lake increases, the co-tidal lines still radiate from a no-tide point, but they become slightly curved; the lines of equal rise and fall become somewhat more complicated in form than the system of concentric circles which constitute such lines for the small body.

If the oscillations of a simple deep lake are so complicated, it is easy to see that those of an irregular-shaped ocean must be extremely complex. In general, however, when such an irregularly shaped surface is set into periodic vibration, it breaks up into component parts, each part vibrat-

ing by itself; most of the ocean's surface is covered by one or more such systems.

The number of the systems and the location of the nodal lines or points will vary with the period of the vibration, or with the length of the wave. When the wave-length is short as compared to the dimensions of the vibrating surface, then the area can oscillate in an infinite number of ways; but when the length of the wave is of the same order of size as the dimensions of the surface, then the division is simple and the nodal lines few.

Now the tidal forces have a period of twelve hours and the wave-length of the corresponding vibration in the deep waters of the ocean is from 5000 to 6000 miles. The areas in the ocean which vibrate in this period will, therefore, be comparatively few, and it is not impossible to locate them approximately. In these areas the vibrations will be large and the resulting tides considerable, for the free period of each approximates closely to the period of the forces. The tides in each area are caused by the oscillations of the waters in that area; they are to a great extent independent of the tides in an adjacent area. The tides are local phenomena, not a world phenomenon.

The character of the oscillation, and of the tide in each area, is largely determined by its lateral boundaries. Gradually shoaling water or converging shore lines will increase the height of the tide,

and such differences in the shore lines determine whether the eastern or the western end of the area shall have the greater rise or fall. It may so happen that the tide at one point is considerably higher than at others, and in such cases the wave there generated may be propagated as a free wave and travel across adjacent areas, modifying to a greater or less extent the individual tide of that area.

The north Atlantic forms a rough trapezoidal basin, the West Indies and the northeastern coast of South America forming one side; a line running from Cape St. Roque to Cape Finisterre in Spain and touching the western coast of Africa on its way forms a second side; the third side extending from Spain to Ireland, thence to Iceland and Greenland; the fourth side being formed by the coast of America from Greenland to Florida. This area breaks up into three vibrating zones, the first of which takes in all that portion of the ocean which lies north and east of a line joining Newfoundland with the Cape Verde Islands and the coast of Africa. This area is about one half wave-length long, and oscillates about a nodal line extending from the Banks of Newfoundland to the north-western coast of Ireland; the high tides at Greenland will be six hours later than the tides on the coast of Africa and Spain. The second area joins this first at right angles off the coast of Africa and forms the shorter arm of a broken cross, the coast of South America being the southern

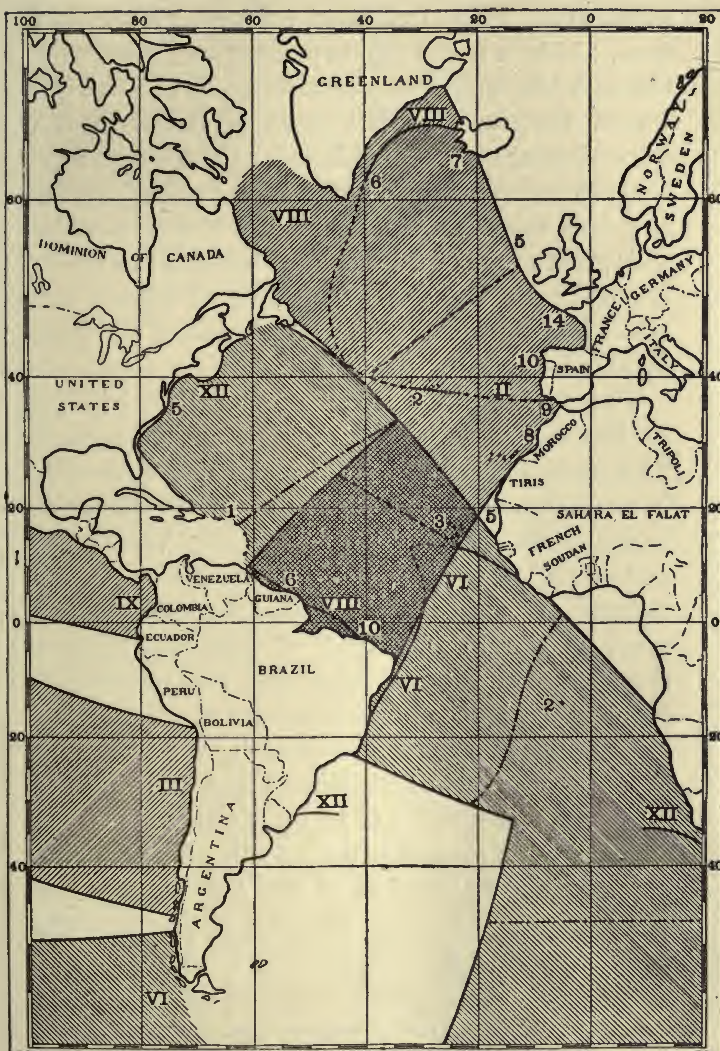

FIG. 25.—The Oscillatory Areas of the North Atlantic

The Roman numbers (XII) indicate the Times of High Water; the Arabic numbers (5) show the mean Rise and Fall of the Tide

boundary. This vibrates in unison with the first area, about a nodal line extending north-westerly through the middle of the ocean. It will be high water, therefore, on the coast of Brazil at the same time as at Greenland, and, at the juncture of the two areas, off the coast of Morocco and Portugal, the range of tides should be considerable. This great range of tides creates a free wave which travels northward along the shores of France and Great Britain, modifying and increasing the pure oscillating tides of those regions.

The third area extends from the United States to the Cape Verde Islands and overlaps the second. This area oscillates about a nodal line extending in a northeasterly direction from the West Indies to the middle of the Atlantic near the Azores. It will be high tide along the American coast at the moment of low tide at the eastern boundary of the area; and from the coast of Newfoundland to the shores of Florida, high and low waters will occur at the same absolute moments.

The tides in the area covered by the two oscillating systems will be confused and the times of high and low water will be intermediate between the times due to each system by itself.

Now this explanation of the Atlantic tides as being due to vibrating motions of more or less definite areas agrees fairly well with the actual conditions. Tidal observations are confined to the coasts and to outlying islands; no data are at hand for drawing the co-tidal lines or for determining

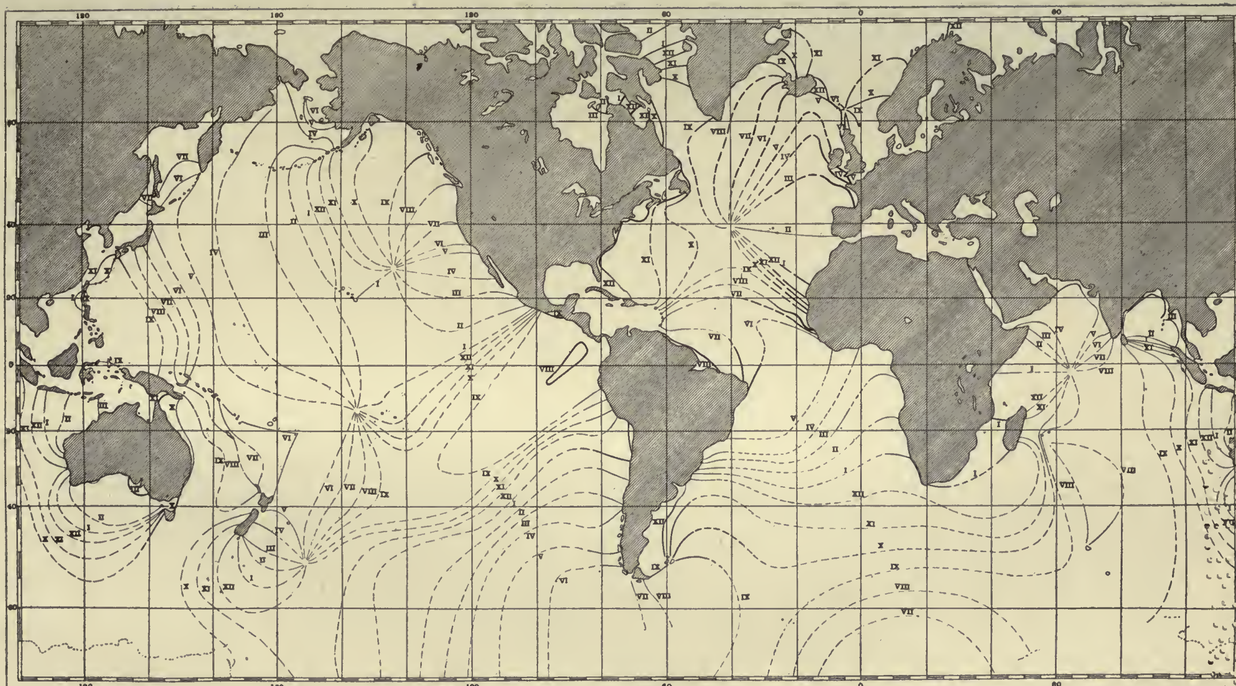

Plate XI

Cotidal Lines of the World, as drawn by Dr. Rollin A. Harris, of the U. S. Coast and Geodetic Survey

the range of the tide in the middle of the ocean. The high tides on the coast of Brazil occur at the same time as do the tides on the coast of Greenland and Iceland, while the tides of Morocco and Spain are six hours earlier. Again there is an abrupt change in the co-tidal lines off the Banks of Newfoundland, and there is some evidence of the existence of the nodal line off the coast of Ireland. In the southern area, on account of the overlapping systems and of the adjacent area of the South Atlantic system, the nodal line is almost completely obliterated.

Diametrically opposed to these ideas developed by Dr. Harris, through the facilities afforded by our Coast and Geodetic Survey, is the generally accepted theory of the tides as a world phenomenon. Newton, La Place, and a succession of brilliant mathematicians have all added the prestige of their names to the latter idea.

Sir George Darwin considers the great earth tides as formed in the broad, deep waters of the southern Pacific. From here the tidal wave spreads east and west, around Cape Horn and past the Cape of Good Hope, and sweeps northward through the Atlantic at a rate depending solely upon the depth of the water. The wave is about forty hours old when it reaches the eastern coast of the United States, and nearly sixty when it arrives at London and the ports of the German Ocean. "The tides along our coast [Great Britain] to-day are mainly due to the action of the moon,

yesterday and the day before, upon the waters of the Indian and Pacific Oceans." In support of this theory, Darwin reproduces a chart published by Airy some seventy years ago, and on this chart are drawn the co-tidal lines, stretching across the Atlantic and showing a seeming progression of the tidal wave from the south northward.

Now fortunately there is a way in which these two opposing tidal theories may be tested. Twice each month, at the times of new and full moon, the tidal wave is greater than at other times. If, therefore, the tidal wave originates in the southern Pacific, the spring tides should occur in the Pacific and Indian oceans some forty hours earlier than in the northern Atlantic. The interval between the conjunction of sun and moon and the occurrence of springs at any point should show a regular progression from the Pacific around Cape Horn and up the Atlantic seaboard. Yet no such progression is found. Spring tide occurs along our coast from Halifax to Old Point Comfort almost simultaneously with the springs in the Indian Ocean, nearly fourteen hours *before* springs at Cape Horn and the Cape of Good Hope, and over sixty hours before that at Melbourne. Along the Pacific coasts of North and South America the time of springs shows not the slightest trace of progression. At San Francisco and the California coast generally springs follow conjunction by from four to nine hours, at Port Townsend by twenty-four, at Panama by

fifty-seven, at Valparaiso by twenty-one, and at Cape Horn by forty-three. The entire coast seems to be broken up into four or five regions, in each of which the time of springs is approximately the same for all points; but in passing from one region to another the time changes abruptly.

The evidence would seem to be in favour of the new idea that the tides are local in their origin.

CHAPTER XI

THE PREDICTION OF THE TIDES

THE tides are local phenomena, they are caused primarily by successive actions of the tidal forces upon local oscillating systems. Each ocean basin is divided into local oscillating areas, each area having a free period approximately equal to twelve hours, the period of the forces, and the oscillations or tides in each area are nearly independent of those in other portions of the globe. In some cases, however, a large oscillation gives rise to a free wave, which traverses adjacent areas and modifies their individual oscillations and causes complicated and irregular tides. The wave from the Pacific spreads east around Cape Horn and sweeps northward up the Atlantic. Here it meets the oscillations of the southern Atlantic, modifies them, and changes the simple tides along the extreme southern coast of South America. This modifying effect of the Pacific wave, however, dies out long before the north Atlantic is reached. In fact the mean range of tide at Rio Grande do Sul is only about 4 inches. The Atlantic tide

becomes apparent at Rio de Janeiro, where the mean range is $2\frac{1}{2}$ feet; at Pernambuco the mean rise and fall is 5 feet.

While the tides are thus local in character and origin, differing in every portion of the globe, yet the ultimate cause of the tides, the tidal forces, are independent of local conditions. These forces are astronomical, depend upon the relative positions of the earth, sun, and moon, and, therefore, the tidal periods are the same the world over. In periodicity, in time intervals, the tides are world phenomena; in character and in the actual rise and fall of the waters, they are local.

The tidal forces due to the attraction of the moon have been discussed. The sun acts in precisely the same manner, although its effects are much smaller. The tidal forces decrease as the cube of the distance between the earth and the disturbing body increases; and thus, although the sun is immensely greater than the moon, yet its greater distance more than overbalances its great mass, and the solar tide-generating force is only about two fifths that of our small satellite. When the moon is directly overhead a 20,000-ton steamer loses five pounds of its weight; when the sun is directly overhead such a steamer loses two pounds only. When the two bodies, the sun and the moon, lie on the same or opposite sides of the earth, when it is new or full moon, the two forces will act together, and a 20,000-ton steamer, because of the existence of moon and sun, will lose

seven pounds. When the moon is in quadrature, the moon being overhead and the sun on the horizon, the actions of the moon and sun partially offset one another, and the steamer will lose between three and four pounds only. In exactly the same way the horizontal or tide-producing forces, due to the sun and moon, conspire at new and full moon and counteract each other at quadratures. These horizontal forces are of about the same magnitude as the vertical, and hence at new and full moon the tidal forces are very much larger than at the time of first and third quarters. The oscillations of the waters are more or less proportional to the forces and thus at new and full moon large tides or springs are formed; while at first and last quarter the tidal range is only about one third of the springs, and these small tides are the so-called neaps. The tidal curves in Fig. 20 agree very well with this; the spring and neap tides occur within a day or two of the changes of the moon and the spring range is a little over three times that of the neap. On the coasts of the United States, the neap range generally exceeds one half of the spring range in magnitude.

Again, the sun is on the equator on but two days a year. At all other times, therefore, the solar tidal forces will have a diurnal inequality, similar to that of the lunar forces. Each body produces two distinct tides a day, a daily tide and a semi-daily tide. And the tidal days of the two bodies differ by fifty minutes; the solar tidal day

being exactly twenty-four mean solar hours long, the lunar tidal day being twenty-four hours and fifty minutes. The semi-diurnal solar tide has a period of twelve hours; the semi-diurnal lunar tide a period of twelve hours and twenty-five minutes. The diurnal inequality in the actual tide is thus caused by several partial diurnal inequalities in the forces. They affect in nearly opposite ways the two high waters and the two low waters of a day.

The semi-diurnal portion of the lunar or solar tide at any point of the earth's surface is, in general, the more important. It is practically independent of the declination of the sun or moon, and to it the tides are principally due. The diurnal portions of both tides, however, vary almost directly with the declinations of the sun and moon, vanishing when those bodies are on the equator, and reaching a maximum when they are at their greatest distances north or south of the equator. The moon is on the equator twice each lunar month, the sun twice each year. Thus the diurnal inequality in the height of the tide will not only vary from day to day each month, but also from month to month during each year.

The relative size and position of the diurnal component with respect to the semi-diurnal determines the type of the tide. And their relative sizes and positions are mainly influenced by local conditions. The tide in any portion of the ocean is seldom purely oscillating—the oscillation

is modified by progressive waves. In island regions the progressive wave becomes especially prominent and the reflections and interferences of the tidal waves explain the great variety of tides there found. An island, a reef, or other obstruction affects a short-period wave to a far greater extent than a long-period one. A group of islands may completely obliterate a short-period vibration, while hardly affecting a long wave. Thus in island regions the diurnal wave becomes prominent and the tide may be almost purely diurnal.

The character of the tide at any one place may be described by "tidal constants," which consist of certain time intervals and certain heights. Those generally used in tidal tables and works on navigation refer in some way to high and low water and are the so-called non-harmonic constants. The establishment of the port, to which reference has already been made, is one of the most important of these. These constants can only be found by observation, but when once found they may be used to predict future tides.

The corrected establishment and the average range of the tides are, of course, the most important of these constants. The corrected establishment, or the "high water interval" (HWI) is the average number of hours and minutes which elapse between the transit of the moon (upper or lower) and the next succeeding high water. As a rule the low waters do not follow the highs by an interval of exactly six hours and twelve minutes,

and for this reason there is a "low water interval" (LWI) which corresponds to the high water interval and represents the average number of hours and minutes between the time of transit and the time of low water. The actual intervals on any given day will seldom equal these average values, for during a part of each month the crest of the solar wave will be in advance of the lunar wave, and the tides will occur earlier than usual. This effect is called the "priming" of the tide. Similarly during the remainder of the month the solar wave causes the interval to be larger than usual, and there is said to be a "lagging" of the tide.

The average difference in height between high and low water is called the "mean range" (Mn). The actual difference in the height of the tides varies greatly, being greatest at springs and least at neaps. The average of the springs is called the "spring range" (Sp), and the average of the neaps is called the "neap range" (Np).

As the depth of the water is constantly varying it is necessary to adopt some standard plane to which the height of the tide at any time can be referred. The natural plane for this purpose would be either the half-tide level, or the mean sea-level. These two differ very little from each other; the half-tide level, as its name suggests, is a plane half-way between the average heights of all the high waters and of all the low waters. To obtain the mean sea-level the height of the

tide is measured every hour, either on a staff gauge or from the curve drawn by an automatic register. The average of all these hourly readings fixes the mean sea-level, and this would exactly agree with the half-tide level if the duration of the rise and fall of the tide be exactly the same. For practical reasons, however, these natural planes of reference are not used. On charts it is desirable that the soundings shall show the smallest depth of water that a navigator may expect to find, and for this reason a low stage of the tide is adopted. All the charts of the Atlantic and Gulf coasts, as prepared by the Coast and Geodetic Survey, give the depth of the water when the tide is at the average height of all the low waters. This plane of reference is called that of "mean low water," and it is obviously one half of the mean range (Mn) below the half-tide level. At extreme low tides the water will fall below this level, but only by a small amount. At Sandy Hook the tide fell below the plane of reference on 135 days during the year 1908, but the extreme low tide was only one foot below the plane.

After the high water interval, or the corrected establishment of the port, and the mean range of the tide, the most important tidal constants have reference to the diurnal inequality. This difference in the heights of the two tides each day is caused by the diurnal component, which reaches its maximum value when the moon is farthest from the equator. The tides at these times are officially

spoken of as "tropic tides," because the moon is then near one of the tropics, and the then value of the diurnal inequality is called the "tropic diurnal inequality." The tropic inequality in high waters (HWQ) is the difference in height between the two high waters in a given day at this period; the tropic inequality in low waters (LWQ) is the corresponding difference in the height of the two low waters.

As these two inequalities are caused by the diurnal wave, the range of this wave can be found from their values. An approximate expression for this range is given by

$$\sqrt{(HWQ)^2 + (LWQ)^2}$$

and this is the maximum value of the diurnal wave near the time the moon is farthest from the equator. From the Shelter Island curves it is readily found that, when the moon is farthest north or south of the equator, the higher high tide exceeds the lower high tide by about 0.9 foot, while the lower low tide is only 0.2 foot lower than the higher low tide of the same day. The first of these is the "tropic inequality in high waters"; the second the "tropic inequality in low waters." From the two the maximum range of the diurnal wave is found to be a trifle over 0.9 foot. As the mean range at this station is only 2.5 feet, the diurnal wave is nearly one half the size of the semi-diurnal. The greatest range of the tides at this time, that is, the difference between the high and the lowest low water, if

consecutive, is called the "great tropic," and at Shelter Island this was found to be 2.9 feet. All along the Atlantic coast of the United States, from Cape Cod Peninsula to Key West, Florida, this ratio between HWQ and LWQ is approximately the same; that is to say, LWQ is much smaller than HWQ. All along the Pacific coast, from Mexico to Bering Sea, inclusive, the diurnal wave is generally much larger than upon the Atlantic coast; and here, too, the inequality in the low waters considerably exceeds that in the highs.

The presence of the diurnal wave not only changes the heights of the water, but it also alters the high and low water intervals. The time at which high water occurs will depend upon the relative positions of the two waves; highest high water will occur at some time intermediate between the passages of the crests of the semi-diurnal and diurnal waves. The greatest change in the high and low water intervals will be noticed, of course, during the tropic tides, at which time these intervals are usually redetermined and are called respectively "tropic higher high water" and "tropic lower low water" intervals. At Shelter Island these intervals differ from the mean intervals by five minutes and twenty minutes respectively.

All these tropic inequalities are shown on the following diagram, which represents the tropic tides at San Francisco, where the diurnal component is relatively much larger than at Shelter Island.

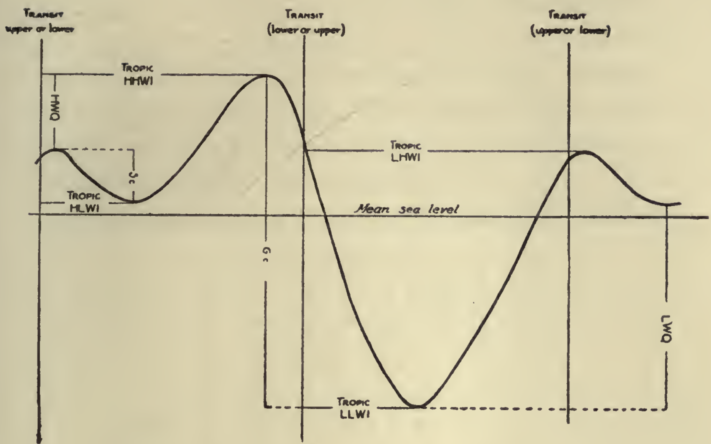

FIG. 26.—Non-Harmonic Tidal Constants

These various constants can be fairly well determined from one month's consecutive observations of the high and low waters, and when so determined they can be used to predict future tides. For this purpose it is necessary to take from the almanac the times of upper and lower transits of the moon, its phases and times of greatest and least distances from the equator. By adding to the transit times the proper intervals the times at which high and low waters should occur can be found within a few minutes, and from the ranges can be made an approximate estimate of the height of the tide. The tide tables annually published by the Coast and Geodetic Survey give the values of these constants for many hundreds of harbours throughout the world, and

from them the tides for any day can readily be found.

Among the earliest known tide tables is one which was written in the thirteenth century and contained in a manuscript belonging to the Abbey of St. Albans. It consists of a single leaf giving "flod at London brigge," in the following manner:

Aetas Lunae.	*h.*	*m.*
1	3	48
2	4	36
3	5	24
4	6	12
—	—	—
—	—	—
28	1	24
29	2	12
30	3	0

In this little table the time increases by the constant difference of 48 minutes each day. It shows that at the time of full and new moon it was high tide at 3^h 48^m after the moon's transit. The present value of this interval is about 2 hours, showing thus that the changes in the river during the past centuries have altered the tidal constants by a considerable amount.

Flamsteed, the first Astronomer Royal, observed the tides and published a corrected table for the year 1683 giving the times of both high waters of the day at London Bridge. He also collected data for the tides in other ports and published a

supplemental table giving the tidal differences by which the times of the tides in various ports could be found from the table of the London tides. For the most part, however, the early tables for Liverpool, London, and other ports were constructed by private individuals, who kept their methods secret and who resented scientific investigation and the publication of scientific and official tables. These tables were practically all made from an analysis of long series of observations, and some of them were remarkably accurate.

In 1867 Sir Wm. Thomson (Lord Kelvin) devised a most powerful method for analysing tidal observations. This method, called "harmonic analysis" has been enlarged and perfected by Sir Geo. H. Darwin, who has placed tidal observations and predictions on the highest scientific plane. On its face this method is extremely elaborate and laborious, but it is now used, whenever the observations cover a sufficient period to render exact work advisable.

The method depends upon the well-known principle that any periodic motion, or oscillation, no matter how complicated it may be, can always be resolved into the sum of a series of simple circular, or harmonic, motions. This principle, now used in all branches of physical research, was probably invented by Eudoxas as early as 356 B.C., when he explained the apparently irregular motions of the planets by combinations of uniform circular motions. This rough explanation

of Eudoxas was afterwards elaborated by Hipparchus and Ptolemy into the complete and consistent "epicyclic" theory of planetary motions.

Suppose a particle to revolve at a uniform speed in a circular path, then if we viewed this motion from a point in the plane of the circle, we would see the particle apparently oscillate up and down through a vertical path. Near the middle of its straight course the particle would apparently move faster than when near either end. In fact, as it approached the highest point of its path the particle would rise very slowly, then stand still for an instant, and finally begin to fall, at first slowly, then more and more rapidly as it approached the middle part of its path. This to-and-fro oscillation, or simple harmonic motion, is well illustrated by the ordinary pendulum, the motion of which is visibly quickest at the middle of its course and which slackens as it approaches either end of its swing.

Simple harmonic motion, then, is motion which is periodic, which is a to-and-fro oscillation along a line, and which may be studied by comparison with uniform motion around a circle. The length of the swing of the particle from its middle position, or the "amplitude," is equal to the radius of the circle; the interval of time required for the particle to make one complete journey back and forth is the "period"; and the "phase" is that fractional part of a period which has elapsed since the particle last passed through the upper point of its course.

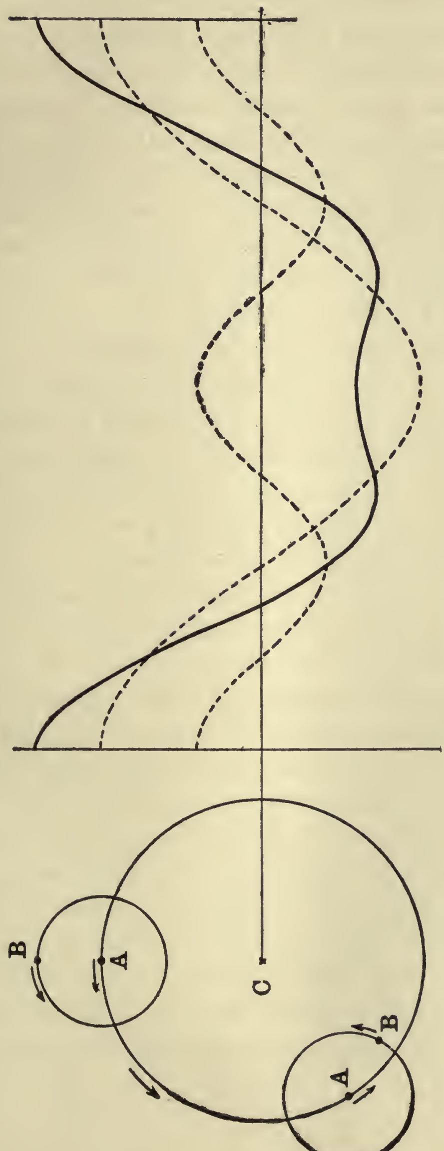

FIG. 27.—Composition of Harmonic Motions.

Now if, as our particle A revolves in its circle, a second particle B revolves about it in a smaller circle at twice the speed of A, then the actual motion of this second particle will be rather complicated. Suppose that when A is at the upper point in its course, B is also at the highest point on the small circle: then they will both be moving in the same direction and B will actually be moving with a speed equal to the sum of the individual speeds of both particles. When, however, A has passed over a certain portion of its path, it may be falling, while B, having moved more rapidly in its small circle, will be rising, and rising more rapidly than A is falling, so that at such a time the actual motion of B will be upward. When A reaches the lowest part of its course, B will have completed a revolution in its path, and be again at the highest point of its small circle; it will, therefore, not fall as low as does A. Viewed edge-on the actual motion of B would thus appear extremely irregular: it would first fall rapidly, then stop, rise for a moment, then fall again, to its lowest point, then rise, fall, and finally return to its starting-point. This apparently capricious motion is seen to be the result of two independent simple harmonic oscillations.

By changing the relative amplitudes, periods, and phases of the two component motions, the resultant apparent motion of B can be made to assume a great variety of forms. So long, however, as the two periods are commensurable, the

resultant motion will be periodic, and the particle will finally return to its starting-point, to begin over again its erratic journey. If the periods be not commensurable, then the particle can never return exactly to the starting-point, no matter how many oscillations there may be, and the actual path never can become a closed curve. By the addition of a third harmonic motion, the actual path of the particle may be still further complicated.

In order more easily to study the motion of such an oscillating particle, it is usual to draw the motion out into the form of a curve, by supposing the particle to have an additional uniform motion in a line at right angles to the line of oscillation. Such curves are the tidal curves, which have already been studied at some length. The pencil which rises and falls with the water, represents the harmonically oscillating particle, while the forward motion is given to the paper by clock-work, and thus a continuous curve is drawn.

Now, at any port, the rise and fall of the water under the action of the tidal forces is periodic and it may, therefore, be separated or analysed into a number of harmonic components. Each component tide will be simple, will have a fixed and definite period and a constant amplitude. This method of separating the tide into a number of partial constituents was first suggested and made practical by Sir Wm. Thomson a little more than a quarter of a century ago. It is independent of any theory as to the cause of the tides; it is equally

applicable, whether the tides be a great progressive wave, or a local up-and-down oscillation. The tide at each port is treated by itself, is analysed separately, and the resulting constituent tides apply to that port and to that port only.

The periods of these partial tides, however, are the same for all ports, for these periods depend upon the periodic variations in the tide-producing forces. The actual values of these forces at any one time depend, as we have seen, upon the relative positions of the sun and moon; these forces are astronomical in origin and vary with the motions of the sun and moon. These motions have been studied for centuries, have been analysed, and are known with the highest degree of precision. So accurately are these motions known, that the lengths of the component periods in the tide-generating forces can be calculated to within a thousandth part of a second.

To fully determine the character of a simple oscillation, and to predict the future position of the moving particle, three elements must be known: the period, the range, and the phase of the oscillation at a given moment. Now the periods of all the component tidal oscillations are known, but the range and the phase of each oscillation can only be determined from observation at each port. The range determines the height to which the water will rise; the phase, the time at which the component high water occurs.

The complete analysis of the tide at any one

port shows the presence of some twenty or more component oscillations. For each oscillation two quantities, the range and the phase, must be determined. These quantities are the "harmonic constants," and must be found from local observations for each harbour. When high precision is required, therefore, some forty or more constants must be found, and in order to find them a long series of observations, extending over years, is necessary. Most of these oscillations, however, are extremely small as compared to the six or eight principal components, and thus for ordinary practical work it is only necessary to consider some eight partial tides, and to find some sixteen constants for a given port.

The analysis of the tides and the disentanglement of the separate oscillations is a complicated and tedious process. The oscillations all coexist and the height of the water at any given instant is a result of all the component tides. The periods of the partial tides differ, however, and this difference of period furnishes the key to the analysis. Consider for a moment the two principal tides—the mean lunar and the solar semi-diurnals; these are ordinarily denoted by M_2 and S_2, respectively, the subscript "2" signifying that the component to which it pertains is semi-daily in character. The lunar tide has a period of $12^h\ 25^m\ 14\frac{1}{6}^s$; the solar of exactly 12^h. If the solar tide alone existed, the water would be at the same height at corresponding morning and afternoon hours of

each and every day. If at noon and midnight of any one day, for example, the water were found to be six inches above half sea-level, then at noon and midnight of every day it would be exactly the same height, six inches. But, if the lunar tide alone be considered, the result is different; the water reaches the same level at intervals of $12^h\ 25^m\ 14^s$. So if the water be one foot above the mean at noon of one day, it will have a different height, either greater or less, at noon of the next day. The height of the water at noon on different days will vary, thus, from the highest to the lowest; on the average the height above mean sea-level will be zero. In a long series of noon observations the effect of the lunar tide disappears. Thus with the two tides existing at the same time, the average of a large number of noon observations will give the height of the water at that time due to the solar oscillation alone. Similarly for each and every other hour of the day: the average height of the water will be due to the solar tide alone. With these hourly heights of the solar tide, the range and the interval from noon up to high water can easily be found and these are the two tidal constants for the solar semi-diurnal oscillation.

By a similar process the tidal constants for the lunar oscillation can be found. The period of this partial tide is $12^h\ 25^m\ 14\frac{1}{6}^s$, and at succeeding intervals of this length the water will be at the same height. By taking the average height of the water

at intervals of this length, therefore, the effect of the lunar wave can be separated, and the lunar constants found. This necessitates measuring the height of the water twenty-four times each day, and each measurement will occur at some odd minute and second, instead of at the even hours, as in the case of the solar component. By like systems of averaging, each and every other constituent oscillation might be singled out and its corresponding two constants determined. But the work of analysing the tidal observations of a single port for even one year would be so enormous as to be practically impossible of accomplishment.

Fortunately there are modifications of this method by which the work can be shortened and the tidal constants approximately determined from the measured height of the water at successive mean solar hours. The number of measurements is thus reduced to twenty-four in each day—a number which can be handled without making the work so large as to be prohibitive.

When the tides at a given port have been thus analysed and the harmonic constants found, the prediction of the tides for that port becomes a simple matter. As each partial tide repeats itself with unfailing regularity, the height of the water due to each constituent tide at any instant can readily be calculated. The solar tides repeat themselves day after day at the same hour, the lunar tides at the same lunar hour or at the same interval

after the moon crosses the meridian. The almanac gives the time of the moon's transit, and it is a simple matter to find at any time the lunar hour and thence the height of the lunar tide. The actual height of the water will be the sum of the heights of the partial tides. This tabulation of the heights of the water due to each component tide is a perfectly simple operation, but it is quite laborious, if all the partial tides are taken into account. It is purely mechanical, and in fact tidal predictions are now nearly all made by machine: a machine invented by Sir Wm. Thomson replaces the computer and does the work accurately and in far less time.

Although in actual construction the machine is complicated, yet in principle it is extremely simple. It reproduces on a small scale the actual up-and-down oscillations of a water particle, moving under the action of the tidal forces. To show how this can be done, consider the weight W supported at the end of a cord which passes over the pulley A and the other end of which is fastened at F. The pulley may be given an up-and-down motion by means of a crank and pin C. If C rotates uniformly the motion of the shaft and of the pulley A will be harmonic. The weight W will, therefore, move up and down harmonically and its total range will be four times the length of the crank at C. If now a longer cord be substituted—one that will pass around a fixed pulley at W, then over a second movable

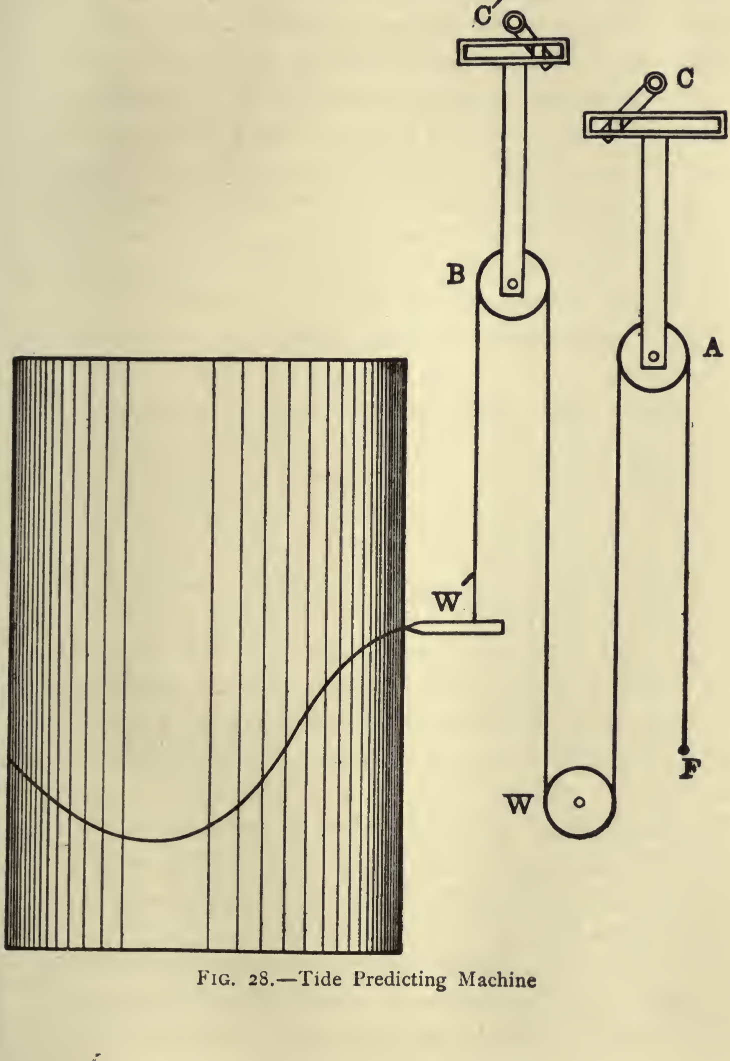

FIG. 28.—Tide Predicting Machine

pulley at B, finally ending in a weight at W′, then the motion of W′ will be the sum of the motions of the two pulleys A and B. If A and B both move up and down harmonically, W′ will oscillate harmonically and its motion will be the combined motions of the two harmonic constituents A and B. The lengths of the crank arms represent the amplitudes of the component oscillations; the time of revolution of the wheels, the periods; and the relative directions of the crank arms at a given moment the relative phases of the waves.

The lunar day is $24^h\ 50^m\ 28^s$ long, or it is 1.035050 solar days. If, therefore, the two crank wheels are geared together in such a way that A completes its oscillation faster than B, and the two periods are in this definite ratio, then the motion of A may represent a solar tide, while B represents a lunar component. Further, let the lengths of the crank arms be adjusted so that each shall be $\frac{1}{48}$ part of the solar and lunar diurnal ranges respectively, and the directions of the arms be such as to indicate the relative phases of these two components; then as the crank wheels revolve the weight W′ will oscillate up and down, reproducing, on a scale of one inch to the foot, the motion of the water due to these two partial tides.

In order to make a permanent record the weight may be replaced by a pencil which presses against a revolving drum. If this drum be connected with

the gear wheels, so that it makes only one revolution while A makes two, then each revolution of the drum will represent one day, and the pencil will trace out upon it the daily tidal curve.

In the actual machine it is necessary to have a crank arm and pulley to represent each component tide. The first machine, made in 1876, contained ten components; later machines provide for fifteen and twenty partial tides. The machine used by the Coast and Geodetic Survey, and designed by Wm. Ferrel, provides for nineteen components and gives directly the times and heights of high and low waters. In this machine the curve is not actually drawn and no computations are required. In order to predict the tides for a given year and place, it is necessary to adjust the lengths of the arms so that each shall be the same proportion of the known height of the corresponding partial tide. Each arm must also be set at the proper angle to represent the phase of the component at the beginning of the year. When all these adjustments have been made, it takes only a few hours to run off the tides for a year.

The Coast and Geodetic Survey publishes annually tidal predictions for many harbours. On the Atlantic and Gulf coasts, from Newfoundland to Galveston, they give the complete daily predictions for twenty prominent ports. For each of these places the tables give the times of each high and low water during the year, together with

the heights of the sea at those instants. For more than one thousand additional ports on this coast the Survey publishes tables of tidal constants and tidal differences, from which the times of high and low waters can be found with very little calculation. The tidal differences are merely the average differences in time and in range between the tides at the given port and those of one of the twenty standard ports, for which complete prediction is made. For example, Governor's Island, New York, is one of the standard ports, and to it are referred the tides in New York and Newark bays, and the Hudson River to Albany. At Yonkers the tidal differences are $+57^m$ and $+64^m$ respectively; that is, on any given day high water at Yonkers will follow high water at Governor's Island by 57^m, while low water will be delayed $1^h\ 4^m$. At Yonkers, also, the range of the tide is some seven inches less than at New York, the high tide being that much lower than the corresponding tide at Governor's Island.

CHAPTER XII

TIDES AND TIDAL CURRENTS OF THE ATLANTIC COAST

THE ocean tides are mainly oscillations, the water in the great basins swinging to and fro under the action of the generating forces of the sun and moon. These simple oscillations or standing waves are, however, greatly modified by shoaling water and the bounding shore lines. In many cases local progressive waves are formed, which travel across adjacent oscillating areas, modifying, confusing, and sometimes almost obliterating the original simple oscillation. But in general the deep-water tides are simple as compared to those in the bays and harbours of the coast.

The costal tides are local oscillations or progressive waves set up and caused by the tides of the deep ocean. Only indirectly are they due to the tidal forces. The bays and basins of the coast are so small that the direct tides set up in them by the tidal forces are absolutely inappreciable. Long Island Sound, for example, is prac-

tically an enclosed basin about one hundred miles long, and the direct tide in a lake of this length would be less than $\frac{2}{3}$ of an inch in height. The actual tides in the Sound vary from three to seven feet, and they cannot, therefore, be directly due to the tidal forces of the sun and moon. Nor, on the other hand, can they be due to a simple progressive wave derived from the sea, for they are much larger than the deep-water tide, and the times of high and low water are nearly the same throughout the length of the Sound.

The exact boundaries where the deep-water oscillations give place to the coastal tides are of course vague and ill-defined. Generally, however, they should be drawn free from capes and headlands to points where the shore suddenly takes another trend, or to where outlying shoals interfere with the free oscillations of the waters. Some of the coast waters thus consist of great wide open bays, others of deep narrow channels, or of broad sounds connected with deep water by narrow mouths.

Now when the waters at the mouth of such a bay or channel are periodically disturbed by the ocean tides, what sort of a disturbance is propagated into the bay? The case is simply another example of free and forced vibrations already fully discussed, and the character of the oscillation into which the waters of the bay are thrown will depend upon its length and the depth of its waters. As has been seen, the velocity with which a long

wave travels depends solely upon the depth of the water, and hence in shallow waters a free tidal wave moves slowly, and its length is short in comparison with the length of the wave in the open sea.

If the length of the canal or bay be very short in comparison to the length of a free tidal wave in its waters, then the waters of the canal will simply rise and fall as a whole, keeping in unison with the waters of ocean. Tides are simultaneous, throughout the length of the canal, it being high water within at the same time it is high water without. The flood will begin at low tide, ebb at high, and the velocity of the flood and ebb will be greatest near the mouth. At the head of the canal the waters will rise and fall without any horizontal motion, there being no current at this point.

If the canal be more than ¼ wave-length long, a nodal line will cross the canal at a distance of ¼ wave-length from the head or closed end; the waters of the canal or bay will oscillate about the nodal line, it being low water at the head of the canal at the time it is high water at the mouth, and in the open sea. The height of the tide will be the greatest at the ends of the canal, while near the nodal line the range will be very small. Should the length of the canal be a multiple of ½, then it will break up into a series of stationary waves separated by nodal lines. At stations distant one, two, and three wave-lengths from the mouth of

the canal, it will be high water at the same time it is high water outside; at intermediate stations it will be high water when it is low water outside.

A more important case, however, is that in which the canal or bay is approximately one quarter wave-length. Such a canal will interfere with and disturb the deep ocean oscillation near the mouth of the canal. This is easily seen by considering the motion of a simple progressive wave to and fro in the canal. At the time of high water in the ocean just outside the canal, a progressive wave will start up the canal, will reach the head at the end of three hours, be reflected and returned to the mouth. This reflected high water will reach the mouth at the time of low water outside and will be propagated into the ocean, where it will meet the incoming high tide. At the mouth the reflected high water will neutralise the low due to the ocean oscillation, and the water will stand at mean level. The succeeding ocean high at the mouth will similarly be neutralised by the canal; an approximate nodal line will be established, and the waters within the canal will finally oscillate as a standing wave, the phase of the oscillation being three hours, or ¼ period, after the corresponding phase outside. Just outside the approximate nodal line, the simple oscillation of the ocean will be disturbed by the reflected wave and the phase of the oscillation will change rapidly from point to point. In fact, in this part of the ocean the wave will be

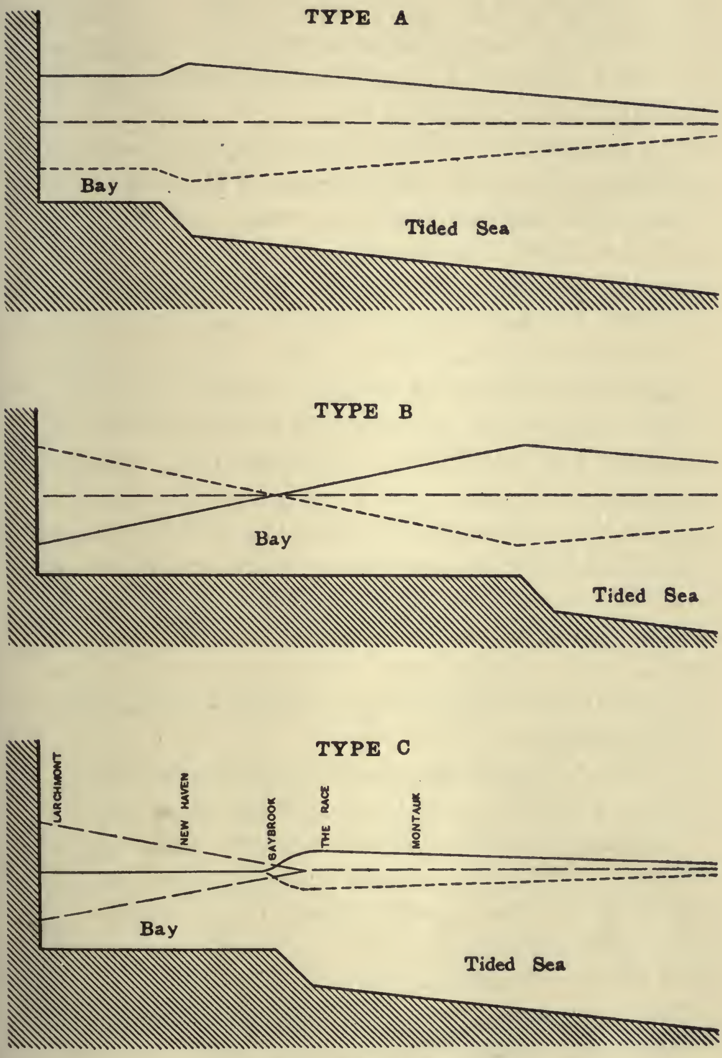

FIG. 29.—Types of Bay Tides

progressive. Thus the ocean tidal oscillation, through an intermediate progressive wave, supports in the canal an oscillation, whose phase is delayed one quarter period.

These three cases are shown in the foregoing diagram, in which the positions of the waves are shown in section at different stages of the tide. In the third case, shown as Type C, the nodal point will not be sharply defined, for even at it the waters will have a rise and fall somewhat equal to the rise and fall of the ocean. Further, as the tides inside are three hours later than those outside, there will be more or less confusion near the mouth and very abrupt changes in the co-tidal hours.

In the case of a long river the tide takes the form of a simple progressive wave which passes up the river at a rate due solely to the depth. The up-stream flood is strongest at the time of high tide. Such river tides are found in the Hudson, Potomac, James, and other tided rivers of the coast.

Now to apply these general principles to the tides of the Atlantic Coast, it must be noted that the great Atlantic basin extends from Nova Scotia to Florida and its boundaries are nearly parallel to the coast. From Cape Sable the boundary line curves to the south and west outside of Georges Bank and the shoals off Nantucket; thence nearly touches the shores of Martha's Vineyard, making another southward bend off

Montauk, after which point it runs practically parallel to the coast all the way to Florida. At all points of this line high water occurs at practically the same instant and the deep-sea oscillations are nearly at right angles to it. The tidal currents will thus set back and forth across this line, the flood beginning soon after low water and reaching its greatest velocity three hours later. At the time of high water the particles will have reached their greatest western elongation and the ebb will begin.

To the north of the boundary line are the Gulf of Maine and the Bay of Fundy. The Gulf of Maine is a comparatively shallow rectangular basin, about 250 miles long from Nantucket to Nova Scotia and some 200 statute miles deep from Georges Bank to Seguin. The Bay of Fundy is a narrow inlet about 150 miles deep, leading off from the north-east corner of the gulf. At the outer boundary of the gulf the waters are about 100 fathoms deep; near the coast they are from 50 to 60 fathoms; the average depth is probably not far from 75 fathoms. For this depth of water the length of a free wave having a period of twelve lunar hours would be somewhere near 1000 statute miles, and therefore a quarter wave-length would be about 250 miles. The Gulf of Maine, therefore, satisfies very nearly the conditions of the third general case discussed above; from mouth to head it is about one quarter wave length. Near the boundary line, from

Georges Bank to Nantucket Island there should be more or less of a nodal line, while within the gulf there should be a standing wave increasing in height as the coast is approached. At all points within the gulf, high water should occur at practically the same instant, and this time should be about three hours later than high water outside in the deep ocean. Near the nodal line the times of the tides should change very rapidly; three co-tidal lines being very close together.

Now this approximates very closely to the actual conditions. Co-tidal lines stretch from Nova Scotia to Nantucket, and at either end of these the rise and fall of the tide is small. At Nantucket the mean range is but two (2) feet, while at Sable Island it is a trifle over three (3) feet. Within the gulf from Quoddy Head to Cape Cod there is but half an hour's difference in the times of high water, while the range increases from four (4) feet off Chatham to eleven (11) feet at Bar Harbor.

The Bay of Fundy is a narrow arm of the Gulf of Maine and the tides therein are part of the general gulf tides. It is high water in the bay at the same time that it is high along the Maine coast, but the range of the tides increases rapidly toward the head of the bay. This is in a great measure due to the converging shore lines and to the lessening depth of water. The largest tides of the region occur at the head of the Minas Basin and

on the Petit Condiac River, where the respective ranges are 43 and 40 feet.

Among the islands and in the deep bays of the coast of Maine, the tide is derived from the general tidal wave of the gulf. Up the narrow channels the tide is propagated as a free wave, its speed depending chiefly upon the depth of the water. In its progress the wave is reflected from the sides of the islands and from the head of the bay, and its rate and phase often undergo considerable alterations. The longer the wave, the less such island barriers affect it, and thus the diurnal wave passes in more readily than the semi-diurnal and the type of tide may thus vary rapidly from point to point. In such complicated regions it is almost, if not quite, impossible to predict from theory alone what the tide at any one point will be.

Coming down to regions nearer New York, the tides of Long Island Sound are analogous to those in the Gulf of Maine. Long Island Sound from the meridian 71° 45′ W., which passes a few miles to the eastward of Montauk Point, to Execution Rock is 108 statute miles, while the average depth is 12 fathoms. In water of that depth, the length of a free wave having a period of 12 lunar hours would be 406 statute miles. Hence from the meridian 71° 45′ W. to Execution Rock is almost exactly one quarter wave-length, and the waters of the Sound itself should oscillate as a whole, high water occurring throughout its length

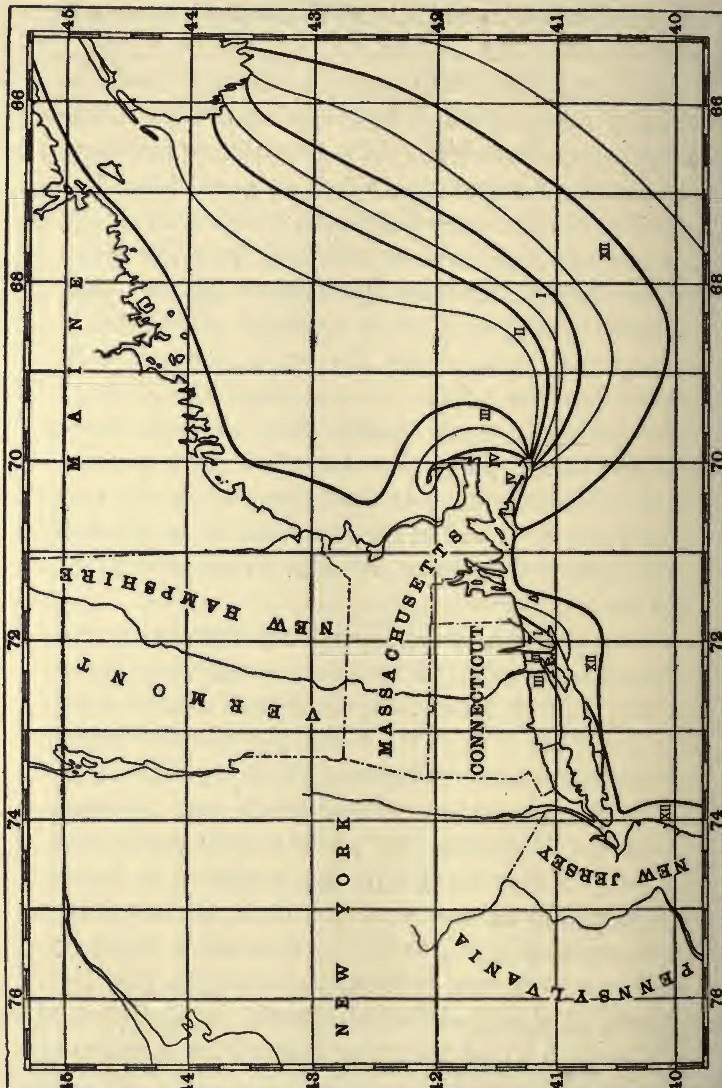

FIG. 30.—Co-tidal Lines of the New England Coast
(U. S. Coast and Geodetic Survey.)

some three hours after high water outside, and the range of the tide should increase towards the western end. Now this is what is found: at Block Island and on the south shore of Long Island, the high water interval (or time of high water after moon's transit) is about $7\frac{1}{2}$ hours; while within the Sound, from Saybrook to Execution Light, the average interval is $10\frac{3}{4}$ hours. From Block Island to Saybrook the intervals change very rapidly, the co-tidal lines running in a north and south direction and being very close together. Again, from Block Island to Saybrook the tidal range is from two to three feet; but from Saybrook to Execution it steadily increases until it becomes over seven and a quarter ($7\frac{1}{4}$) feet at the western end of the Sound.

While there may be some little progression in the tidal wave of the Sound, due to the incoming progressive ocean wave which sustains the movement, it is clear from the above facts that the Sound tides are mainly due to a standing wave. For in the waters of the Sound a free progressive wave would travel at the rate of 33 miles per hour, and it would therefore take such a wave somewhat more than two hours to travel a distance of 75 miles, from Saybrook to Execution. Now the tide at Execution is only 36 minutes later than at Saybrook, only about one quarter of the time required for a wave to travel between the two places. Again, if the wave were progressive it should flatten out and the range become less

as it passes into the broad portion of the Sound opposite New Haven, to again become higher and steeper in the narrow western end. But instead of this the range increases uniformly from the east towards the west; off New Haven the range being six feet—over two feet higher than at Saybrook.

The tides in the adjacent bays and harbours, as Peconic, Jamaica, and Great South bays, are chiefly due to hydraulic effects. When the water is higher in the ocean, it flows in through the narrow mouths, fills the bays until the waters within and without are at the same level. As the waters in the ocean fall, the waters within are at a higher level and a strong stream or current will flow out until the level is once more restored. In these bays, therefore, the tides are always later than those outside and the range is less. At Jamesport, at the head of Peconic Bay, the tide is three hours later than at Montauk Point, and the range is slightly less than it is in Gardiner's Bay and Block Island Sound.

The general ocean tidal oscillation approaches the coast very closely off Sandy Hook, but the gradually shoaling water and the converging shore lines cause a somewhat larger range than is found farther to the eastward. The tide proceeds through the bay and up the Hudson River in the form of a progressive wave, which throughout the harbour has about the same range as at Sandy Hook. At Governor's Island and

the Battery the tide is about half an hour later than at the Hook, at West 96th Street about one hour, at Ossining a little over two hours, at Poughkeepsie four hours and twenty minutes, and at Albany nearly ten hours later. As the wave proceeds up the river it moves thus more and more slowly, and its height gradually diminishes until at Albany the rise and fall is only a little over two feet.

Tidal Currents

In the deep ocean the tidal forces cause, as has been seen, a to-and-fro oscillation of the water about a nodal line. High water occurs simultaneously throughout one half of the oscillating area, and at the same moment it is low water at all points on the other side of the line of no rise and fall. The separate particles of water all oscillate in unison, each describing twice over in twelve hours a rectilinear path slightly inclined to the horizon. When they reach the westward end of their swing, it is high tide in the westward half of the vibrating area and low tide in the eastern. At this moment the velocity of the particles is zero, and it is slack water throughout the area. As the particles start on their eastern swing, ebb begins at the western and flood at the eastern end, and three hours later, or at the time of mean sea-level, the particles are all moving to the eastward with their maximum velocity, and the ebb and flood have their respective maximum

intensities. Hence, at either boundary of a simple ocean area, flood begins at low tide, and reaches its maximum three hours before high water. Slack occurs at high and low waters respectively.

In such oscillating areas the maximum velocity of the water particles at the nodal line will vary with the depth of the basin, being greater the shallower the basin. In the ocean basins the maximum velocity of the ocean particles at the nodal line is less than $\frac{1}{10}$ foot per second for each foot of the semi-range of the tide at the boundaries. If the basin be of uniform depth, bounded by abrupt shores, then the on- and off-shore velocity of the water particles would be zero at the land and would increase according to a simple law from the shore to the above maximum at the nodal line. But if the basin had a bottom sloping uniformly downward from the shores, then the on- and off-shore velocity of the particles would be practically the same for a considerable distance outward from the shore line.

When such an oscillating area is bounded by a shoal, or island obstructions, a progressive wave is generally created. Now in a wave of progression the particles describe distinctly elliptical paths in vertical planes, and the maximum forward velocity of each particle is at the moment it reaches the top of its path. In such a wave the flood stream begins three hours before high water, reaches its maximum velocity at high water. Slack water occurs at half tide level and the ebb begins three

hours after high. The actual velocity of the water particle depends upon the height of the wave and the depth of the water, being greater the higher the wave and the shallower the water. For a tide having a semi-range of two feet in water thirty (30) feet deep the maximum velocity of a particle is a trifle over two (2) feet per second, or the tidal current would have a maximum velocity of 1.2 miles per hour. The currents in tidal rivers are of this character, but are modified by the permanent downward current of the river.

Near the coast the actual tide is often a combination of a stationary oscillation and a progressive wave. When two such currents cross, the resulting current is peculiar and frequently shows circular points, about which the currents revolve.

Well off our eastern coast the tidal current is an on- and off-shore oscillation and the strength of the flood is three hours before high water. The tidal hour (*i.e.* the Greenwich lunar time of local high water) in the deep waters off the Gulf of Maine is XII, and the hour of maximum current is found to be about IX½. Between the ocean tide and the true gulf tide, there is a narrow region off the Georges Bank where the intermediate progressive wave predominates. In this region the current hour changes rapidly from IX½ to XII½ and the current is much stronger than farther outside.

Within the gulf the wave is stationary in character and the tidal currents consist of an

on- and off-shore oscillation, the maximum of the flood occurring throughout the entire gulf at practically the same moment, some three hours before high water. From Georges Bank to the Bay of Fundy the time of the current changes by less than one half hour. Throughout the chief part of the gulf the current is relatively weak, but, owing to the converging shore lines, it increases in strength as the Bay of Fundy is approached. Along the shores of the bay the current turns earlier, because the wave motion of the water is partially destroyed by the irregularities of the shore.

In the narrow passages near the head of the bay the currents become very strong. As has been noted, the mean range of the tide is here nearly 40 feet and in consequence the water level changes very rapidly. In the passage to the Minas Basin the current often reaches six knots, while near Cape Split eight knots have been recorded.

In the harbours and along the narrow passages between the islands of the Maine coast, the currents are mostly due to difference of level. As the tide rises off the coast, the water at the mouth of the channel is higher than that within and a current flows in until both are at the same level. Again the tide falls more quickly in the open, and a current runs out until equilibrium is again restored.

To the eastward of Nantucket, on the Georges

Bank the currents are due to the intermediate progressive wave. The flood sets into the Gulf of Maine, the ebb flows from it, and the currents are rather strong, reaching at times three knots. The current hours increase rapidly in a northerly direction, from IX in the deep sea to XII in the gulf, just north of the Georges. To the south and west of Nantucket a similar progressive wave travels westward toward Long Island Sound, and the current hours change rapidly towards the west. Where these two currents, the northerly and the westerly, divide, a circular point is formed, and thus from a point somewhere near the Nantucket Light Vessel the co-current lines radiate like the spokes of a wheel. To the eastward of this point the hours increase to the northward, to the west the progression is to the south and westward.

In Vineyard Sound the tides are principally due to the wave coming in from the south and westward, while in Nantucket Sound the tide comes from the east. The flood passes Gay Head and in through the Muskeget Channel at about the same time, and the two reach Cross Rip together. In Vineyard Sound, from Gay Head to Nobska, the strongest set of the easterly current is found at local high water. Thus the current sets eastward for three hours of the rising and three hours of the falling tide. At Cross Rip there is a region where the current hour is the latest and where it is nearly one hour later than at Gay

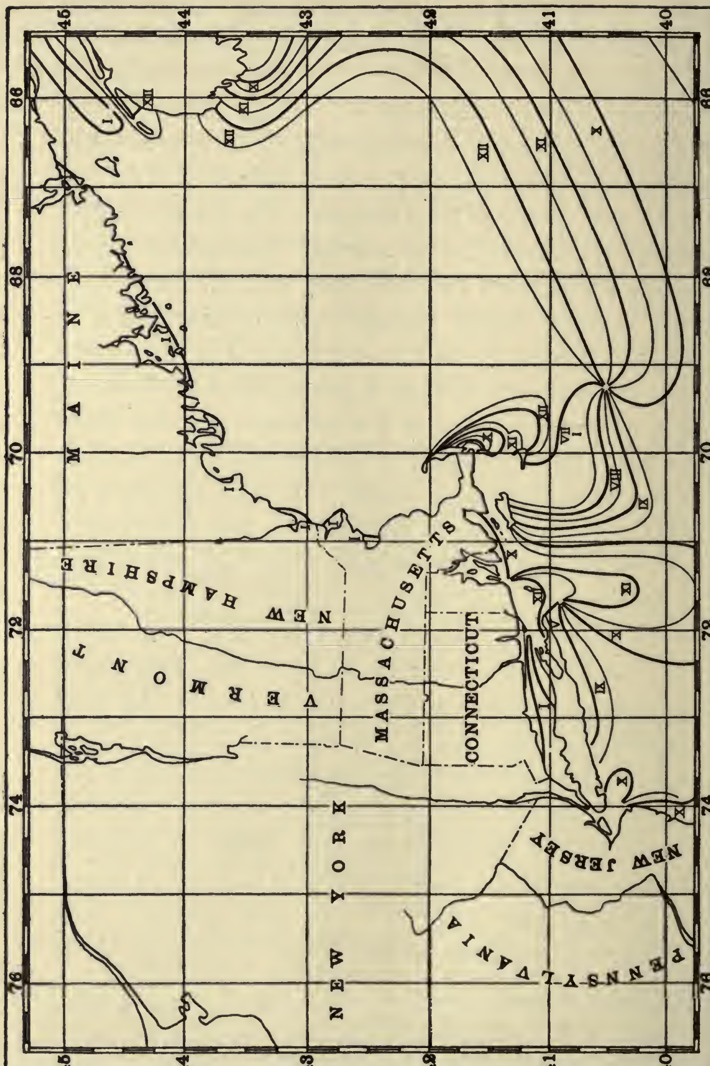

FIG. 31.—Co-current Lines of the New England Coast
(U. S. Coast and Geodetic Survey.)

Head. To the eastward of this region the current turns earlier, until off the northern point of Nantucket, where the current hour again increases, but the direction of the current changes to the northward. This change to the north and east of Nantucket is caused by the fact that the tides of the Gulf of Maine predominate in this region. Along the eastern shore of the Monomoy Peninsula, and extending well up towards Chatham, there is a region in which the current turns about 2½ hours earlier than it does to the eastward of Great Point. In fair weather, therefore, it is advisable for a vessel, with an adverse current, to keep well in under the shore, for she will get the change of current some hours before a vessel farther out at sea.

Off Point Judith there is another circular point, the current dividing, part passing up into Narragansett Bay and part westerly into Block Island Sound. To the east of this point the current hours increase towards the south, between the circular point and Narragansett Pier they increase to the north. In running west from Newport, therefore, with a favourable current it is advantageous to stand well off shore, for by so doing a vessel will keep the current longer. With an adverse current a vessel should keep well under Point Judith, for she would here get the turn of the current nearly four hours before a vessel off shore.

As has been seen, the tide in Long Island Sound

is mainly stationary, and the maximum current will be, therefore, some three hours before high and low waters, and the current will turn simultaneously throughout the length of the Sound. It is slack water at both high and low water. The current hour in the axis of the Sound is about one hour later than that to the eastward of Block Island: that is, in passing from Block Island to the Race the current hour increases slightly. The passage-way to the Sound is obstructed by Fisher's, Plum, and Gull islands, and these obstructions necessitate a greatly increased velocity in the currents though the intervening passages. In the Race the velocity is three knots, while in the main part of the Sound it is barely one. Along the shores of the Sound the current turns earlier than in the axis, and this is especially noticeable along the north shore and in Fisher's Island Sound, where it turns nearly an hour and a half before it does in the Race and in the main body of the Sound.

Tables

TABLE I

LENGTH OF A DEGREE (60′) OF LONGITUDE IN DIFFERENT LATITUDES

Lat.	Miles.	Lat.	Miles.	Lat.	Miles.	Lat.	Miles.	Lat.	Miles.
0°	60.00	20°	56.40	40°	46.03	60°	30.08	80°	10.45
1	59.99	21	56.04	41	45.35	61	29.16	81	9.42
2	59.96	22	55.66	42	44.66	62	28.24	82	8.38
3	59.92	23	55.26	43	43.95	63	27.31	83	7.34
4	59.85	24	54.84	44	43.23	64	26.37	84	6.29
5	59.77	25	54.41	45	42.50	65	25.42	85	5.25
6	59.67	26	53.96	46	41.75	66	24.47	86	4.20
7	59.55	27	53.50	47	40.99	67	23.51	87	3.15
8	59.42	28	53.02	48	40.22	68	22.54	88	2.10
9	59.26	29	52.52	49	39.44	69	21.56	89	1.05
10	59.09	30	52.00	50	38.64	70	20.58		
11	58.90	31	51.48	51	37.83	71	19.59		
12	58.70	32	50.93	52	37.02	72	18.60		
13	58.47	33	50.37	53	36.19	73	17.60		
14	58.23	34	49.79	54	35.34	74	16.59		
15	57.97	35	49.20	55	34.49	75	15.58		
16	57.69	36	48.60	56	33.63	76	14.56		
17	57.39	37	47.98	57	32.75	77	13.54		
18	57.08	38	47.34	58	31.87	78	12.51		
19	56.75	39	46.69	59	30.98	79	11.48		

TABLE II

Dip and Distance of Horizon for the Average State of the Atmosphere.
Minus to Observed Altitude

Height of Eye.	Dip.		Distance Miles.	Height of Eye.	Dip.		Distance Miles.
0	0'.00	0' 00"	0.00	35	5'.82	5' 50"	6.79
1	0.98	0 59	1.15	36	5.90	5 54	6.89
2	1.39	1 23	1.62	37	5.99	5 59	6.98
3	1.70	1 42	1.99	38	6.07	6 04	7.08
4	1.97	1 58	2.30	39	6.14	6 08	7.17
5	2.20	2 12	2.57	40	6.22	6 13	7.26
6	2.41	2 25	2.81	41	6.30	6 18	7.35
7	2.60	2 36	3.04	42	6.38	6 22	7.44
8	2.78	2 47	3.25	43	6.45	6 27	7.53
9	2.95	2 57	3.44	44	6.53	6 32	7.62
10	3.10	3 06	3.63	45	6.60	6 36	7.70
11	3.26	3 15	3.81	46	6.67	6 40	7.79
12	3.40	3 24	3.98	47	6.75	6 45	7.87
13	3.55	3 33	4.14	48	6.82	6 50	7.95
14	3.68	3 40	4.30	49	6.89	6 53	8.04
15	3.81	3 49	4.45	50	6.96	6 58	8.12
16	3.94	3 56	4.59	55	7.30	7 18	8.52
17	4.06	4 03	4.73	60	7.62	7 37	8.89
18	4.17	4 10	4.87	65	7.93	7 56	9.26
19	4.29	4 17	5.00	70	8.23	8 14	9.61
20	4.40	4 24	5.13	75	8.52	8 31	9.94
21	4.51	4 31	5.26	80	8.80	8 48	10.27
22	4.62	4 37	5.39	85	9.07	9 04	10.59
23	4.72	4 43	5.51	90	9.33	9 20	10.89
24	4.82	4 49	5.62	95	9.59	9 35	11.19
25	4.92	4 55	5.74	100	9.84	9 50	11.5
26	5.02	5 01	5.85	120	10.78	10 47	12.6
27	5.11	5 07	5.97	140	11.64	11 38	13.6
28	5.21	5 13	6.08	160	12.45	12 27	14.5
29	5.30	5 18	6.18	180	13.20	13 12	15.4
30	5.39	5 23	6.29	200	13.92	13 55	16.2
31	5.48	5 28	6.39	300	17.05	17 03	19.9
32	5.57	5 34	6.50	400	19.68	19 40	23.0
33	5.66	5 40	6.60	500	22.00	22 00	25.7
34	5.74	5 44	6.69	600	24.10	24 06	28.1

Note.—Abnormal Refraction may change these values.

TABLE III

CORRECTION TO DIP, AS GIVEN IN TABLE II, FOR ABNORMAL REFRACTION DUE TO DIFFERENCE OF SEA AND AIR TEMPERATURES.

Plus (+) to Dip, when *Air* is *cooler*.
Minus (−) to Dip, when *Air* is *warmer*.

Diff. of Sea and air. Fahr.	Height of Eye.			
	0 Feet.	20 Feet.	40 Feet.	60 Feet.
1°	0′ 11″	0′ 25″	0′ 31″	0′ 35″
2	0 23	0 36	0 43	0 48
3	0 34	0 48	0 55	1 0
4	0 46	1 0	1 7	1 13
5	0 57	1 11	1 19	1 25
6	1 8	1 23	1 31	1 37
7	1 20	1 35	1 43	1 50
8	1 31	1 47	1 55	2 2
9	1 43	2 0	2 9	2 15
10	1 54	2 12	2 19	2 27
20	3 48	4 6	4 19	4 31
30	5 42	6 3	6 19	6 35
40	7 36	8 0	8 19	8 39

This table, reprinted from Inman's *Nautical Tables*, depends upon observations and is not to be considered as universally applicable. It affords some indication of what to expect in calm weather, when the temperature of the air an_ sea differ.

TABLE IV

Corrections for Refraction, Parallax, and Semi-Diameter to be applied to the Observed Altitude of the Sun's lower limb, and the correction for Refraction to be applied to the Observed Altitude of a Star, to obtain the True Altitude.

Barometer, 30 inches; Thermometer, 50° F; Sun's Semi-Diameter 16′ 00″.

Observed Altitude.	Correction. Sun. +	Correction. Star. −
5° 00′	6′ 15″	9′ 54″
10	6 31	9 38
20	6 46	9 23
30	7 00	9 9
40	7 14	8 55
50	7 27	8 42
6 00	7 39	8 30
10	7 51	8 18
20	8 2	8 7
30	8 13	7 56
40	8 24	7 45
50	8 34	7 35
7 00	8 44	7 25
10	8 53	7 16
20	9 2	7 7
30	9 10	6 59
40	9 19	6 50
50	9 27	6 42
8 00	9 34	6 35
10	9 42	6 27
20	9 49	6 20
30	9 56	6 13
40	10 2	6 7
50	10 9	6 0
9 00	10 15	5 54
10	10 21	5 48
20	10 27	5 42
30	10 33	5 36
40	10 38	5 31
50	10 44	5 25
10 00	10 49	5 20
10	10 53	5 15
20	10 58	5 10
30	11 2	5 6
40	11 7	5 1
50	11 12	4 56
11 00	11 16	4 52
10	11 20	4 48
20	11 24	4 44
30	11 28	4 40
40	11 32	4 36
50	11 36	4 32

Observed Altitude.	Correction. Sun. +	Correction. Star. −
12° 00′	11′ 40″	4′ 28″
10	11 44	4 24
20	11 48	4 20
30	11 51	4 17
40	11 54	4 14
50	11 58	4 10
13 00	12 1	4 7
10	12 4	4 4
20	12 7	4 1
30	12 10	3 58
40	12 13	3 55
50	12 16	3 52
14 00	12 18	3 50
10	12 21	3 47
20	12 24	3 44
30	12 26	3 42
40	12 29	3 39
50	12 31	3 37
15 00	12 33	3 35
20	12 38	3 30
40	12 43	3 25
16 00	12 47	3 21
20	12 51	3 17
40	12 55	3 13
17 00	12 59	3 9
20	13 3	3 5
40	13 7	3 1
18 00	13 10	2 58
20	13 13	2 55
40	13 17	2 51
19 00	13 20	2 48
20	13 23	2 45
40	13 26	2 42
20 00	13 29	2 39
20	13 32	2 36
40	13 35	2 33
21 00	13 37	2 31
20	13 40	2 28
40	13 42	2 26
22 00	13 44	2 24
20	13 47	2 21
40	13 49	2 19

Observed Altitude.	Correction. Sun. +	Correction. Star. −
23° 00′	13′ 51″	2′ 17″
20	13 53	2 15
40	13 55	2 13
24 00	13 57	2 11
20	13 59	2 9
40	14 1	2 7
25 00	14 3	2 5
30	14 6	2 2
26 00	14 9	1 59
30	14 12	1 56
27 00	14 14	1 54
30	14 17	1 51
28 00	14 19	1 49
30	14 21	1 47
29 00	14 23	1 45
30	14 25	1 43
30 00	14 26	1 41
30	14 28	1 39
31 00	14 30	1 37
32 00	14 34	1 33
33 00	14 37	1 30
34 00	14 41	1 26
35 00	14 44	1 23
36 00	14 47	1 20
37 00	14 50	1 17
38 00	14 52	1 15
39 00	14 55	1 12
40 00	14 57	1 10
41 00	14 59	1 7
42 00	15 1	1 5
43 00	15 3	1 3
44 00	15 6	1 0
45 00	15 8	0 58
46 00	15 10	0 56
47 00	15 12	0 54
48 00	15 13	0 53
49 00	15 15	0 51
50 00	15 17	0 49
51 00	15 19	0 47
52 00	15 20	0 46
53 00	15 21	0 44
54 00	15 23	0 42

Observed Altitude.	Correction. Sun. +	Correction. Star. −
55°	15′ 25″	40″
56	15 26	39
57	15 27	38
58	15 29	36
59	15 30	35
60	15 31	34
61	15 32	33
62	15 33	31
63	15 34	30
64	15 36	28
65	15 37	27
66	15 38	26
67	15 39	25
68	15 40	24
69	15 41	22
70	15 42	21
71	15 43	20
72	15 44	19
73	15 45	18
74	15 46	17
75	15 47	16
76	15 48	15
77	15 49	13
78	15 50	12
79	15 51	11
80	15 52	10
81	15 53	9
82	15 54	8
83	15 54	7
84	15 55	6
85	15 55	5
86	15 56	4
87	15 57	3
88	15 58	2
89	15 59	1

TABLE V

Corrections for the Variation in the Sun's Semi-Diameter

January	+15″	July	−15″
February	+11″	August	−11″
March	+ 5″	September	− 5″
April	− 4″	October	+ 4″
May	−11″	November	+11″
June	−16″	December	+16″

TABLE VI

The following tables give the principal dimensions and elements of the Solar System, together with data concerning the physical characteristics of the eight larger planets.

(A) Dimensions and Elements of the Solar System

	Name.	Semi-Major Axis of Orbit.	Mean Dist. Millions of Miles	Sidereal Period (mean solar days).	Period in Years.	Orbit-Velocity (miles per second).	Eccentricity.	Inclination to Ecliptic.	Longitude of Ascending Node.	Longitude of Perihelion.	Epoch.
Terrestrial Planets	Mercury . . .	0.38710	36.0	87.9692	0.24	23–35	0.20561	7° 0′ 10″.4	47° 8′ 45″.4	75°53′ 58″.9	1900 Jan. 0.0
	Venus......	0.72333	67.2	224.7008	0.62	21.9	.00682	3 23 37 .1	75 46 46 .7	130 9 49 .8	" " "
	The Earth..	1.00000	92.9	365.2564	1.00	18.5	.01675	0 00 00 .0	00 00 00 .0	101 13 15 .0	" " "
	Mars.......	1.52369	141.5	686.9505	1.88	15.0	.09331	1 51 1 .3	48 47 9 .4	334 13 6 .9	" " "
Major Planets	Jupiter.....	5.20280	483.3	4332.58	11.86	8.1	.04825	1 18 41 .8	98 55 58 .2	11 54 26 .7	1850 Jan. 0.0
	Saturn	9.53884	886.1	10759.20	29.46	6.0	.05606	2 29 39 .3	112 20 51 .4	90 6 39 .5	" " "
	Uranus.....	19.19009	1782.8	30685.93	84.02	4.2	.04704	0 46 21 .6	73 29 24 .9	169 2 55 .6	1900 Jan. 0.0
	Neptune...	30.07067	2793.5	60187.64	164.79	3.4	.00853	1 46 45 .3	130 40 44 .0	43 45 20 .2	" " "

(B) Dimensions and Physical Characteristics of the Planets.

	Name.	Apparent Angular Diameter.	Mean Diameter.		Mass.		Density.		Time of Axial Rotation.	Inclination Equator to Orbit.	Oblateness, or Ellipticity.	Gravity at Surface.
			in Miles.	⊕ = 1.	☉ = 1.	⊕ = 1.	⊕ = 1.	Water = 1.				
	Sun.......	32′ 00″	864,750	109.4	1.00	329,390	0.25	1.40	25^d to 35^d	7° 15′	?	27.65
	Moon......	31′ 26″	2,160	0.27	—	$\frac{1}{81}$	0.61	3.40	$27^d\ 7^h\ 43^m$	6 33	?	0.17
Terrestrial Planets.	Mercury...	5″ to 13″	3,000	0.38	$\frac{1}{6,000,000}$	$\frac{1}{18}$	1.12	6.20	87.97 days	?	?	0.43
Terrestrial Planets.	Venus.....	11″ to 67″	7,830	0.97	$\frac{1}{408,000}$	$\frac{1}{1.2}$	0.86	4.80	?	?	?	0.88
Terrestrial Planets.	The Earth.	—	7,913	1.00	$\frac{1}{329,390}$	1.00	1.00	5.53	$23^h\ 56^m\ 4^s.09$	23° 27′.14	$\frac{1}{293}$	1.00
Terrestrial Planets.	Mars......	3″.6 to 24″.5	4,210	0.53	$\frac{1}{3,093,500}$	$\frac{1}{8.1}$	0.72	4.00	$24^h\ 37^m\ 22^s.65$	24 50	$\frac{1}{220}$	0.44
Major Planets.	Jupiter....	32″ to 50″	85,600	10.92	$\frac{1}{1047.355}$	316	0.24	1.33	$9^h\ 50^m$ to $9^h\ 56^m$	3 05	$\frac{1}{17}$	2.65
Major Planets.	Saturn....	14″ to 20″	73,700	8.97	$\frac{1}{3501.6}$	95	0.13	0.72	$10^h\ 16^m$ to $10^h\ 38^m$	26 49	$\frac{1}{9.2}$	1.18
Major Planets.	Uranus....	3″.8 to 4″.1	32,000	4.03	$\frac{1}{22,869}$	15	0.22	1.22	?	?	?	0.91
Major Planets.	Neptune...	2″.7 to 2″.9	35,000	4.40	$\frac{1}{19,700}$	17	0.20	1.11	?	?	?	0.90

TABLE VII

Magnitude, Distance, and Intensity of the Nearest Navigation Stars

This table gives the magnitude, distance, and actual intensity of eleven of the brighter stars. The distances are given in "light years"; the unit of which represents a distance equal to 63,368 radii of the earth's orbit, or of approximately six million million miles. The "intensity" represents the relative amount of light given out by the star and the sun. The table shows that these stars actually emit from two to one hundred and forty-two times as much light as the sun.

Name.	Mag.	R. A.	Decl.	Parallax.	Distance.	Intensity.
α Centauri	0.7	$14^h.5$	$-60°$	$0''.75$	4.2	2
Sirius	−1.4	6 .7	−17	0 .37	8.8	42
Procyon	−0.5	7 .6	+ 5	0 .34	9.5	2
Altair	1.0	19 .8	+ 9	0 .21	15.3	14
Castor	1.5	7 .5	+32	0 .20	16.3	25
Formalhaut	1.0	22 .9	−30	0 .13	25.0	36
Aldebaran	1.1	4 .5	+16	0 .10	32.5	56
Capella	0.1	5 .1	+46	0 .10	32.5	142
Regulus	1.2	10 .0	+12	0 .10	32.5	51
Vega	0.2	18 .6	+39	0 .10	32.5	129
Polaris	2.0	1 .4	+89	0 .06	54.2	93

TABLE VIII

Forty Useful Navigational Stars, with Approximate Mean Times of Meridian Passage.

This table gives the necessary data regarding 40 of the principal stars used in navigation. From this table can be found by inspection the stars that are available on any given date. The last three columns give the approximate dates at which the stars cross the meridian at 9 o'clock in the evening, at midnight, and at 3 o'clock in the morning. Each star culminates approximately 4 minutes earlier each day; or 1 hour earlier every 15 days.

Mag.	Name.		R. A.	Decl.	Meridian Passage. 9 P. M.	Meridian Passage. Midnight.	Meridian Passage. 3 A. M.
2.1	α Andromeda	Alpheratz	$0^h\ 3^m.7$	+ 28°.6	Nov. 7	Sept. 22	Aug. 7
2.5	α Cassiopeiæ	Schedar	0 35 .4	+ 56 .0	" 14	" 30	" 15
2.2	β Ceti	Diphda	0 39 .1	− 18 .5	" 15	Oct. 1	" 16
1.0	α Eridani	Achernar	1 34 .4	− 57 .7	" 30	" 15	" 30
2.2	α Arietis	Hamel	2 2 .1	+ 23 .0	Dec. 7	" 22	Sept. 6
1.9	α Persei	Mirfak	3 17 .9	+ 49 .5	Dec. 26	Nov. 10	Sept. 25
1.1	α Tauri	Aldebaran	4 30 .8	+ 16 .3	Jan. 13	" 29	Oct. 14
0.2	α Aurigæ	Capella	5 10 .0	+ 45 .9	" 23	Dec. 9	" 24
0.3	β Orionis	Rigel	5 10 .2	− 8 .3	" 23	" 9	" 24
1.8	β Tauri	Nath	5 20 .6	+ 28 .5	" 25	" 11	" 27
1.2	α Orionis	Betelguese	5 50 .3	+ 7 .4	Feb. 3	Dec. 19	Nov. 3
−1.0	α Argus	Canopus	6 22 .0	− 52 .6	" 10	" 27	" 11
−1.4	α Canis Majoris	Sirius	6 41 .2	− 16 .6	" 14	Jan. 1	" 16
1.6	ε Canis Majoris	Adara	6 55 .1	− 28 .8	" 19	" 4	" 20
2.0	α^2Geminorum	Castor	7 28 .8	+ 32 .1	" 27	" 13	" 28
0.5	α Canis Minoris	Procyon	7 34 .6	+ 5 .5	Mch. 1	Jan. 14	Nov. 30
1.2	β Geminorum	Pollux	7 39 .8	+ 28 .2	" 2	" 16	Dec. 1
2.2	ι Argus	Tureis	9 14 .7	− 58 .9	" 26	Feb. 9	" 25
2.2	α Hydræ	Alphard	9 23 .2	− 8 .3	" 29	" 11	" 27
1.3	α Leonis	Regulus	10 3 .6	+ 12 .4	Apr. 7	" 21	Jan. 7
2.5	γ Leonis	Algeiba	10 15 .0	+ 20 .3	Apr. 10	Feb. 24	Jan. 9
2.0	α Ursæ Majoris	Dubhe	10 58 .2	+ 62 .2	" 21	Mch. 7	" 20
2.2	β Leonis	Denobola	11 44 .5	+ 15 .1	May 3	" 19	Feb. 1
1.0	α^1Crucis	———	12 21 .6	− 62 .6	" 12	" 28	" 11
1.2	α Virginis	Spica	13 20 .4	− 10 .7	" 27	Apr. 11	" 25
1.9	η Ursæ Majoris	Benetnasch	13 44 .0	+ 49 .8	June 2	Apr. 17	Mch. 4
0.3	α Boötis	Acturus	14 11 .6	+ 19 .7	" 9	" 24	" 11
2.2	β Ursæ Minoris	Kochab	14 50 .9	+ 74 .5	" 19	May 4	" 21
2.8	β Libræ	———	15 12 .2	− 9 .1	" 24	" 9	" 26
2.3	α Coronæ	Alphacca	15 30 .8	+ 27 .0	" 29	" 14	" 31
2.7	β^1Scorpii	Acrab	16 0 .2	− 19 .6	July 6	May 22	Apr. 6
1.3	α Scorpii	Antares	16 23 .9	− 26 .2	" 12	" 28	" 21
2.1	α Ophiuchi	Rasalhague	17 30 .8	+ 12 .6	" 30	June 14	" 29
0.1	α Lyræ	Vega	18 33 .9	+ 38 .7	Aug 15	" 30	May 15
0.9	α Aquilæ	Altair	19 46 .4	+ 8 .6	Sept. 2	July 18	June 3
2.0	α Pavonis	———	20 18 .5	− 57 .0	Sept. 11	July 27	June 11
1.3	α Cygni	Deneb	20 38 .4	+ 44 .9	" 15	" 31	" 16
2.5	ε Pegasi	Enif	21 39 .8	+ 9 .5	Oct. 1	Aug. 15	July 1
1.9	α Gruis	———	22 2 .6	− 47 .4	" 7	" 22	" 7
1.3	α Piscis Australis	Formalhaut	22 52 .7	− 30 .1	" 20	Sept. 4	" 20
2.6	α Pegasi	Markab	23 0 .3	+ 14 .7	Oct. 22	Sept. 6	July 22

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